ESSAYS ON INDUSTRIAL ORGANIZATION: SEQUENTIAL COMPETITION IN DUOPOLY AND REGULATION UNDER INCOMPLETE INFORMATION

By

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To my parents

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This dissertation is a collection of four essays on industrial organization.

The first essay considers a symmetric Cournot duopoly under sequential input choice and examines the scope of strategic behavior. A first mover advantage makes commitment to a relatively expensive input in an initial stage profitable for one firm. The second firm follows by producing in the later stage with the relatively inexpensive input. This leads to asymmetric "leader-follower" equilibria (in pure strategies) in a symmetric game. However, a symmetric equilibrium in mixed strategies is characterized where both firms invest in the relatively expensive input with positive probabilities.

The second essay analyzes a duopoly with two production periods before the market clears. If production is cheaper in the first period, there is a unique subgame perfect Nash equilibrium (SPNE), at which both firms produce in the first period. When cost is slightly higher in the first period, there are two SPNE at each of which one firm is a leader producing in the first period, while the other is a follower producing in the second period. However, if the cost is significantly higher in the first period, there is a unique SPNE, but now both firms produce in the second period.

The third essay considers competition in two part tariffs. It is shown that the firms coexist in equilibrium with positive profit by serving different segments of the market. Also entry into the market is facilitated. Contestability results are reexamined. Under two part tariffs, one potential entrant with zero sunk cost is no longer sufficient to ensure zero profit for the incumbent.

The fourth essay designs optimal regulatory policy in a setting where the firm has better knowledge of demand than the regulator. When the marginal costs decline with output and the regulator collects information directly from the consumers, a regulatory mechanism is proposed which induces truthful revelation as a dominant strategy for every consumer. Efficient prices are always implemented and the firm commands no rents from its superior information.

Strategic interactions among agents with limited market power is the focus of this research.

CHAPTER 1
INTRODUCTION

This dissertation examines strategic interactions among agents in an industry when each has limited market power. In a collection of four essays, four different scenarios are considered. The first three essays are on the theory of duopoly and the fourth essay is on regulation under incomplete information. The first essay analyzes the scope of strategic behavior with sequential input choice. The second essay considers a Cournot duopoly with two production periods before the market clears and endogenizes the order of moves in duopoly games. The third essay is on competitive two-part tariffs and the fourth essay designs optimal regulatory policy in a setting where the firm has better information about demand than does the regulator. In the first three essays, the existence of a small number of firms leads to each firm having limited market power. Also, sequential choices of inputs and outputs provide additional scopes for strategic behavior. In the fourth essay, the regulator has the authority to dictate prices and outputs in the industry. However, the firm enjoys market power through its superior information about demand.

The first essay "Strategic Behavior with Sequential Input Choice" considers a symmetric Cournot duopoly under sequential input choice. A single homogeneous output is produced with two inputs. However input decisions are made sequentially over two periods. In the first period. the firms simultaneously decide on the level of one input. These decisions become common knowledge and then, in the next period, decisions on the second input are made. The market clears at the end of the second period. It is shown that a "first mover advantage" may make commitment to a relatively expensive input in the first-stage profitable for one firm. The second firm follows by producing in the second-stage with relatively inexpensive input. The "first mover advantage" of costly commitment, such as over investment in capital, has been discussed in the commitment literature (Dixit, 1980). It is well known that an established incumbent firm may overinvest in sunk capital to gain some advantage over a potential entrant. However, in the literature, the role of incumbent and entrant has always been exogenously assigned, rather than being determined endogenously from the model. My work significantly differs in that Stackelberg behavior is endogenized through the timing of entry, rather than being arbitrarily assigned as first and second movers. That is, both the firms are symmetric in their choice sets but asymmetric in equilibrium behavior (in pure strategies). However, a symmetric equilibrium in mixed strategies is characterized where both firms invest in the relatively expensive initial period input with positive probabilities.

The second essay "Cournot Duopoly with Two Production Periods and Cost Differentials" treats one theoretical implication of the previous model. Here I consider a Cournot duopoly with two production periods before the market clears. That is, now the output decisions are made sequentially. Saloner (1987) has shown that, if the production costs remain identical over time, then any outcome on the outer envelope of the reaction functions between the Stackelberg outcomes is sustainable as a subgame perfect Nash equilibrium (SPNE). I show that this result crucially depends on the assumption of identical production costs over time. The continuum of equilibria vanishes for any cost differential across periods. If production is cheaper in the first period, then there is a unique SPNE at which both firms produce only in the first period, producing their single period Cournot-Nash quantities. If production cost is slightly higher in the first period, then there are two SPNE. At each, one firm behaves as a leader producing only in the first period, while the other behaves as a follower and produces only in the second period. However, if production cost is significantly higher in the first period, then again, there is a unique SPNE, but now the firms produce only in the second period.

One important feature distinguishing different duopoly models is whether firms move simultaneously or whether they move sequentially. Much of the traditional duopoly analysis has treated this feature as exogenously given. However, whether duopolists play a simultaneous or sequential move game should not be exogenous but should be determined endogenously from the model. This analysis makes a contribution to the literature on endogenous timing in duopoly games. It is shown that the cost differentials across periods determine whether the duopolists move sequentially or simultaneously.

The third essay ″Duopolistic Competition in Two-Part Tariffs″ considers price competition in two-part tariffs. Previous analysis on price competition has been mostly limited to firms choosing uniform prices. I consider this in a market with two identical firms and infinitely many consumers with varying tastes. Robustness of some well known results (derived under uniform pricing) is examined. Under uniform pricing, two firms are sufficient to bring down the equilibrium price to marginal cost and thus ensure zero profit for each. This result is invariant to the assumptions about the timing of price setting; that is, whether or not we consider a sequential or simultaneous price setting model, the outcomes remain the same. However, it is shown that under two-part tariffs with sequential price setting, the firms can coexist in equilibrium with positive profits by serving different segments of the market. The price leader serves the low demanders, whereas the price follower serves the high demanders. The question of entry is examined next. Under uniform pricing any positive entry cost can create a complete entry barrier. It is shown that, under sequential price setting two-part tariffs promote entry into the market, whether or not the entrant is a price leader or a price follower. In fact, when the incumbent is the price leader and the entrant is the follower, entry is facilitated even if only the incumbent has the option of charging a two-part tariff and the entrant does not. In this case, we also reexamine contestability results under two-part tariffs and show that one potential entrant with zero entry cost is no longer sufficient to ensure zero profit for the incumbent. Finally we analyze competition under two-part tariffs with simultaneous price setting. It is shown that, in this case, the outcomes under uniform pricing vis-a-vis two-part tariffs are the same.

The fourth essay "Regulating a Monopolist with Unknown Demand: An Alternative Approach" designs optimal regulatory policy in a setting where the regulated firm has better information about consumer demand than does the regulator. In the literature, Lewis and Sappington (1988) derive the optimal regulatory mechanism when the regulator depends exclusively on the firm for demand information. An alternative approach has been taken in this essay. Here the regulator solicits demand information only from the consumers. When the marginal costs of production decline with output, Lewis and Sappington's approach does not establish efficient prices. A single price is set regardless of demand. This procedure results in a price that exceeds the marginal cost of production for small demand and that falls short of marginal cost for large demand. The firm also commands rents under all but some intermediate levels of demand. It is shown here that the observed inefficiencies can be corrected if the regulator solicits demand information directly from the consumers. A regulatory mechanism is proposed which induces truthful revelation as a dominant strategy for every consumer. Furthermore, efficient prices are always implemented, the firm commands no rents from its superior information, and the regulator's budget is balanced in expectation.

CHAPTER 2
STRATEGIC BEHAVIOR WITH SEQUENTIAL INPUT CHOICE

Section 2.1: Introduction

Traditional economic theory mostly assumes that all inputs are chosen simultaneously. However, in reality, inputs are often chosen sequentially, rather than simultaneously. For example, firms may choose levels of capital prior to choosing levels of labor input. If a firm's rivals can observe its current stage input decision, before making decisions in the next stage, then they can respond to this decision, through their later stage choice variables. Thus, in a oligopolistic market, sequential input choice provides room for strategic behavior during input choice.

In this chapter, we consider a Cournot model of duopoly with sequential input choice and focus on the underlying basis for the strategic behavior. In particular, we investigate the role of technology in determining the basis for strategic behavior. For example, if there is cost complementarity among inputs, investing more in an earlier stage lowers marginal costs in the later stages, giving the firm a cost advantage over its rivals and working as a credible threat. Thus, in this case the "cost complementarity" provides a basis for strategic behavior.

However, "cost complementarity" provides only one basis for strategic behavior. In this chapter, we vary substitutability between inputs over two polar cases: one where the inputs are perfect substitutes and the other where the inputs are perfect complements. Under perfect substitutability, "cost complementarity" cannot form the basis for strategic behavior, since first-stage investment does not have any impact on second-stage marginal cost. In the case of perfect complements, the level of the first-stage input gives a maximum capacity of production. As long as output is produced within that capacity, the first-stage input does not have any impact on second-stage marginal cost. Since, neither perfect substitutability nor perfect complementarity exhibits "cost complementarity," we use them to investigate whether there is any other basis for strategic behavior.

We show that, in the case of perfect substitutes, a "first mover advantage" may make commitment to a relatively expensive input in the first-stage profitable for one firm. The second firm follows by producing in the second-stage with relatively inexpensive input. Thus, a "first-mover advantage" provides an additional basis for strategic behavior, other than "cost complementarity."

In the literature, the "cost complementarity" aspect of strategic behavior has been analyzed by Spencer and Brander (1983). They considered a two stage game where a cost-reducing R&D decision is taken prior to the output decision. They restricted their attention to a symmetric pure strategy equilibrium and this equilibrium is characterized by an inefficiently high level of R&D, higher output and lower profit. However, their model can be interpreted as a sequential input choice model, with "cost complementarity" between the inputs. Essentially, in their model cost-reducing R&D plays the role of the first input. Under "cost complementarity" between the inputs, investing more in the first stage lowers the marginal cost in the second stage. Thus, a sequential input choice model, where "cost complementarity" alone provides the basis for strategic behavior, can be treated as Spencer and Brander's model with the first input playing the role of cost reducing R&D.

The "first mover advantage" of costly commitment, such as over investment in capital, has been discussed in the commitment literature (Dixit, 1980). It is well known that an established incumbent firm may overinvest in sunk capital to gain some advantage over a potential entrant. However, in the literature, the role of incumbent and entrant has always been exogenously assigned, rather than being determined endogenously from the model. Our work significantly differs in that Stackelberg behavior is endogenized through the timing of entry, rather than being arbitrarily assigned as first and second movers. That is, both the firms are symmetric in their choice sets but asymmetric in equilibrium behavior (in pure strategies). However, there is a symmetric equilibrium in mixed strategies where both firms invest in the relatively expensive initial period input with positive probabilities.

The chapter is organized as follows. Section 2.2 describes the model. In Section 2.3, we determine "first mover advantage" as a basis for strategic behavior. Section 2.4 briefly discusses the general case and concludes the chapter.

Section 2.2: The Basic Model

Our framework is a standard Cournot model of duopoly with the assumption of sequential input choice. A single homogenous output (Q) is produced with two inputs, capital (K) and labor (L). In stage one each firm decides on its level of capital. In the second stage each observes its rival's level of capital and then makes its labor choice. The two firms have identical production functions $Q = f(K, L)$. Demand for output is characterized by a linear inverse demand function $P = a - bQ$, $a > 0$, $b > 0$, $Q \leq a/b$. Firms are assumed to be price takers in the factor market. The wage rate, w, and the rental rate, r, are taken as given by the firms.

We restrict ourselves to the subgame perfect Nash equilibria. The second-stage equilibrium is a Nash equilibrium in labor choices L_1^* and L_2^*, given the levels of capital K_1 and K_2 in the first stage, if and only if $\pi_i(L_i^*, L_j^* | K_1, K_2) \geq \pi_i(L_i, L_j^* | K_1, K_2) \quad \forall\, L_i \geq 0,\ i \neq j = 1, 2$

Thus, for each level of capital (K_1, K_2), we can solve for the optimal levels of labor (L_1^*, L_2^*) and express (L_1^*, L_2^*) as functions of (K_1, K_2). We then substitute these into the profit functions and express the profits of each firm as a function of the capital levels (K_1, K_2).

A pair of capital levels (K_1^*, K_2^*) are a subgame perfect Nash equilibrium, if and only if $\pi_i(L_i^*(K_i^*, K_j^*), L_j^*(K_i^*, K_j^*), K_i^*, K_j^*) \geq$

$$\pi_i(L_i^*(K_i, K_j^*), L_j^*(K_i, K_j^*), K_i, K_j^*) \ \forall\, K_i \geq 0, \quad i \neq j = 1, 2$$

This permits us to solve the model by backward induction, first solving for optimal levels of labor, given capital and then determining the equilibrium levels of capital. Corresponding to an equilibrium (K_i^*, L_i^*) $i = 1, 2$, in the input spaces, the equilibrium $Q_i^* = f(K_i^*, L_i^*)$ follows in the output space.

Section 2.3: First Mover Advantage as a Basis for Strategic Behavior

In this section we vary substitutability between inputs over two polar cases: one where inputs are perfect substitutes and the other where inputs are perfect complements. Without loss of generality for the case of perfect substitutes let f(K, L) = K + L and for perfect complements let f(K, L) = min [K, L]. Under simultaneous input choice, if the inputs are perfect substitutes, then the firms always use the cheaper input to produce the entire output. However, with sequential input choice we will show that this is not necessarily true. In the case of perfect complements, we show that the solutions are identical under the two assumptions about timing.

Perfect substitutability allows a firm to produce the entire output using only one of the inputs and, hence, under sequential input choice, producing in stage one, he can ensure a minimum level of output. Since its rival observes its action in the next stage, producing in the first stage gives it an opportunity to behave as a Stackelberg leader. Thus, if the inputs are perfect substitutes, a firm has an advantage in producing in stage one, which is essentially a first mover advantage.

Depending on factor prices, we consider two distinct cases. First, we consider the case where the rental rate (r) is less than the wage rate (w), that is the first-stage input is cheaper than the second-stage input. The reverse situation will be considered next.

In addition to the advantage of moving first, when the first-stage input is cheaper, producing in stage one, gives the advantage of using the cheaper input. Hence, both the firms will try to take advantage of producing in stage one and thus, in fact, produce the entire output only in the first stage, using the cheaper input capital. Neither uses labor, the costlier input, and hence there is a Cournot game only in the first stage input space, implying a Cournot game in output space using the cheaper input capital. If the inputs were chosen simultaneously, then the firms would always use the cheaper input to produce the entire output. Therefore, the outcomes under sequential input choice will be identical with those under simultaneous input choice. We summarize the above discussion in the following proposition.

Proposition 2.1

If the inputs are perfect substitutes and $r < w$, then the solutions to the sequential and simultaneous games are identical.

Next, we focus on the case where the rental rate (r) is higher than the wage rate (w). Under simultaneous input choice, firms will never use the costlier input capital. However, under sequential input choice this may not be the case. Even if the rental rate is higher than the wage rate, a firm still has the advantage of behaving as a Stackelberg leader, when producing in stage one. Since the revenue of the Stackelberg leader is higher than that of the Cournot player, the gain in behaving as a leader might dominate the loss due to using the costlier input. Therefore, if the rental rate is not much higher than the wage rate, there is an incentive to produce in the first stage. However, if the rental rate is much higher than the wage rate, then the gain from being a leader no longer dominates the loss due to use of the costlier input. Therefore, in this case neither wants to be a leader and both behave as followers, producing simultaneously in the final stage using the cheaper input labor.

So, the solutions become identical with the simultaneous input choice case. The following proposition formally summarizes the above discussion.

Proposition 2.2

If the inputs are perfect substitutes and $r > (0.03a + 0.97w)$, then the solutions to the sequential and simultaneous games are identical.

Proof: Given K_i and K_j, the profit function for firm i is given by

$\pi_i = (K_i + L_i)\{a - b(K_i + L_i) - b(K_j + L_j)\} - wL_i - rK_i$

Firm i maximizes π_i with respect to L_i (in stage two).

Then first order conditions imply,

$\partial\pi_i/\partial L_i = 0 \Rightarrow a - 2b(K_i + L_i) - b(K_j + L_j) - w = 0$

$\Rightarrow L_i = (a - w)/2b - [K_i + K_j/2] - L_j/2$

Also, $L_i \geq 0$. Thus, given K_i and K_j, optimal L_i is chosen as

$L_i = \max\{(a - w)/2b - [K_i + K_j/2] - L_j/2, 0\}$

Step 1 If $K_i \geq (a - w)/3b$, then $L_i = 0$ for any level of K_j

Let $K_i = (a - w)/3b + \varepsilon$, where $\varepsilon \geq 0$. First we will show that both L_i and L_j cannot be strictly positive. We will prove it by contradiction. Suppose $L_i > 0$ and $L_j > 0$. Then

$L_i = (a - w)/2b - [K_i + K_j/2] - L_j/2$

$L_j = (a - w)/2b - [K_j + K_i/2] - L_i/2$

$\Rightarrow L_i = [(a - w)/6b - \varepsilon] - K_j/2 - L_j/2$

and $L_j = [(a - w)/3b - \varepsilon/2] - K_j - L_i/2$

$\Rightarrow 2L_i = [(a - w)/3b - 2\varepsilon] - K_j - L_j$

Substituting for L_j we get, $(3/2)L_i = -(3/2)\varepsilon \Rightarrow L_i = -\varepsilon$

Since $\varepsilon \geq 0$, this is a contradiction to $L_i > 0$.

Thus both $L_i > 0$ and $L_j > 0$ cannot happen. Therefore, if $L_i \neq 0$, then the only remaining possibility is $L_i > 0$ and $L_j = 0$. Suppose $L_i > 0$ and $L_j = 0$. Then,

$L_i = (a - w)/2b - [(a - w)/3b + \varepsilon + K_j/2]$

$\Rightarrow L_i = [(a - w)/6b - \varepsilon] - K_j/2$

Now, $L_j = \max\{(a - w)/2b - [K_j + K_i/2] - L_i/2,\ 0\}$

Therefore, $L_j = 0 \Rightarrow (a - w)/2b - [K_j + K_i/2] - L_i/2 \leq 0$

Substituting $L_i = [(a - w)/6b - \varepsilon] - K_j/2$, we get $K_j \geq (a - w)/3b$

Now, $L_j = 0$ and $K_j \geq (a - w)/3b \Rightarrow L_i = (a - w)/6b - \varepsilon] - K_j/2 \leq -\varepsilon$

Since $\varepsilon \geq 0$, this is a contradiction to $L_i > 0$.

Step 2: If $K_i \geq (a - w)/3b$, then for firm j it is optimal to choose $K_j = 0$.

From step 1 it follows that, if $K_i \geq (a - w)/3b$, then $L_i = 0$ for any levels of K_j. Thus it is optimal for firm j to choose $K_j = 0$, since it can produce the desired level of output using the cheaper input labor.

Step 3: If $K_i = 0$, then the optimal K_j is given by

$$K_j = \begin{cases} 0 & \text{if } r > 0.03a + 0.97w \\ Q^L & \text{if } w < r < 0.03a + 0.97w \end{cases}$$

where $Q^L = (a - w - 2r)/2b$

First note that if K_j is restricted to be less than $(a - w)/3b$, then it is optimal to choose $K_j = 0\ \forall\ r > w$. This can be shown as follows.

If $K_i = 0$, then $L_i = [(a - w)/6b - \varepsilon] - K_j/2 - L_j/2$ and

$L_j = [(a - w)/3b - \varepsilon/2] - K_j - L_i/2$ has a unique solution at

$L_i = (a - w)/3b$ and $L_j = [(a - w)/3b - K_j] > 0$

Therefore π_j

$= [(a - w)/3b]\{a - (a - w)/3 - (a - w)/3\} - r\{K_j - w[(a - w)/3b - K_j]\}$

$= [(a - w)/3b][(a + 2w)/3] - w[(a - w)/3b] - K_j(r - w)$

Since $r > w$, π_j is maximized at $K_j = 0$.

Also, π_j (at $K_j = 0$) $= (a - w)^2/9b$

Now suppose K_j is not restricted to be less than $(a - w)/3b$.

If $K_j \geq (a - w)/3b$, then by step 1, $L_j = 0$. Therefore,

$\pi_j = K_j[a - bL_i - bK_j] - rK_j$ and

$\pi_i = L_i[a - bL_i - bK_j] - wL_i$

$\partial\pi_i/\partial L_i = 0 \Rightarrow L_i = (a - w - bK_j)/2b > 0$ if $K_j < (a - w)/3$

$\Rightarrow \pi_j = K_j[a - (a - w - bK_j)/2 - bK_j] - rK_j$

$\Rightarrow \pi_j = K_j\{[(a + w)/2] - bK_j/2\} - rK_j$

$\partial\pi_j/\partial K_j = 0 \Rightarrow K_j = Q^L = (a - w - 2r)/2b$

And $\pi_j(\text{at } K_j = Q^L) = (a - w - 2r)^2/8b$

Also note that $K_j \geq (a - w)/b$ cannot be optimal since

$\pi_j(K_j = (a - w)/b) \leq 0$.

Thus given $K_i = 0$, optimal K_j is either 0 or Q^L, according as

$\pi_j(K_j = 0) \;{}^{>}_{<}\; \pi_j(K_j = Q^L)$

$\Leftrightarrow (a - w)^2/9b \;{}^{>}_{<}\; (a - w - 2r)^2/8b$

$\Leftrightarrow r \;{}^{>}_{<}\; 0.03a + 0.97w$

Step 4 If $K_i \in (0, (a - w)/3b)$, then optimal $K_j \notin (o, (a - w)/3b)$

Suppose optimal $K_j \in (0, (a - w)/3b)$. Then

$L_i = [(a - w)/3b - K_i]$ and $L_j = [(a - w)/3b - K_j]$

$\Rightarrow \pi_j = [(a - w)/3b][a - (2/3)(a - w)/b] - (r - w)K_j - w[(a - w)/3b]$ Since $r > w$, $\pi_j(K_j \in (0, (a - w)/3b)) < \pi_j(K_j = 0)$. This is a contradiction to $K_j \in (0, (a - w)/3b)$

Step 5 If $r > 0.03a + 0.97w$, then in the (K_1, K_2) space there is a unique equilibrium $(K_i = 0, K_j = 0)$

From Steps 2-4 it follows that,

$K_i = 0 \Rightarrow$ optimal $K_j = 0$ [since $r > 0.03a + 0.97w$]

$K_i \geq (a - w)/3b \Rightarrow$ optimal $K_j = 0$

$K_i \in (0, (a - w)/3b) \Rightarrow$ optimal $K_j \notin (0, (a - w)/3b)$

Step 5 follows from the above facts.

Step 6 If $r > 0.03a + 0.97w$, then in the (L_1, L_2) space corresponding to (K_1, K_2), we have $(L_1 = Q^c, L_2 = Q^c)$ where $Q^c = (a - w)/3b$

Since $K_i = K_j = 0$, $L_i = (a - w)/2b - L_j/2$ and $L_i = L_j$.

$\Rightarrow L_i = L_j = (a - w)/3b = Q^c$.

Combining Step 5 and Step 6, it follows that $\exists$ a unique symmetric pure strategy equilibrium $\{(K_i = 0, L_i = Q^c), (K_j = 0, L_j = Q^c)\}$. If inputs were chosen simultaneously, then it is easy to check that $\{(K_i = 0, L_i = Q^c), (K_j = 0, L_j = Q^c)\}$ is the unique equilibrium. Thus, the solutions become identical under the two assumptions. ▲

Next, we turn to the case where the gain in behaving as a leader dominates the loss due to using the costly input. This can happen only when the rental rate is not much higher than the wage rate. The following proposition shows that if the rental rate is greater than but close to the wage rate, then two pure strategy asymmetric equilibria exist. In each, one of the firms behaves as a Stackelberg leader, producing the entire amount in the first stage and the other behaves as a follower, using only labor in the second stage and serving the remaining market. Denote Q^L as the output chosen by a Stackelberg leader and Q^F as the output of a follower.

Proposition 2.3

If the inputs are perfect substitutes and $w < r < (0.03a + 0.97w)$, then the sequential input choice game does not have a symmetric pure strategy equilibrium. There are two asymmetric equilibria as follows:

$\{(K_i = Q^L,\ L_i = 0),\ (K_j = 0,\ L_j = Q^F)\}\quad i \neq j = 1,\ 2$

Proof: Since $w < r < 0.03a + 0.97w$, then from Steps 2-4 of Proposition 2.2 it follows that in the (K_1, K_2) space there are only two corner solutions $(K_i = Q^L,\ K_j = 0)$, $i \neq j = 1, 2$. Then in the (L_1, L_2) space corresponding to $(K_i = Q^L,\ K_j = 0)$ we have $(L_i = 0,\ L_j = Q^F)$, where $Q^F = (a - 3w + 2r)/4b$. Therefore, there are only two asymmetric equilibria $\{(K_i = Q^L,\ L_i = 0),\ (K_j = 0,\ L_j = Q^F)\}\quad i \neq j = 1,\ 2$ ▲

From the above two Propositions 3.2 and 3.3, it follows that a Cournot/ Stackelberg game is played according as $r \gtrless (0.03a + 0.97w)$. Also, it can be noted that, $r \gtrless (0.03a + 0.97w)$ according as $(a + w - 2r)^2/8b \gtrless (a - w)^2/9b$, where $(a + w - 2r)^2/8b$ is the profit of a Stackelberg leader and $(a-w)^2/9b$ is the profit of a Cournot player. Therefore, the nature of the game played depends on whether the profit of a Stackelberg leader is higher than that of a Cournot player or not. In other words, it depends on whether or not the gain from being a leader dominates the loss due to use of the costlier input.

Also, Proposition 2.3 explicitly shows the nonexistence of a symmetric pure strategy equilibrium. This nonexistence comes through the fact that there is no interior solution in the first-stage input space. In the first-stage input space there are only two corner solutions with $K_i = Q^L$ and $K_j = 0$, $i \neq j = 1, 2$. These corner solutions in the first stage generate corner solutions in the second stage with $L_i = 0$ and $L_j = Q^F$ and brings a "leader-follower" relationship, giving rise to an asymmetric solution in the output space.

Combining the above three Propositions 2.1-2.3, it is interesting to note how the nature of the game played depends on the factor prices. If $r \in (w, 0.03a + 0.97w)$, we observe a Stackelberg game, whereas a Cournot game is observed if $r \notin (w, 0.03a + 0.97w)$.[1] Thus, the outcomes under sequential input choice differ from those under simultaneous input choice if $r \in (w, 0.03a + 0.97w)$; that is, the rental rate is higher but not much higher than the wage rate.

If $r \in (w, 0.03a + 0.97w)$, the sequential game yields only asymmetric "leader-follower" solutions (in pure strategies), while the simultaneous game would give a symmetric equilibrium. However, with two identical firms it is very difficult to justify an asymmetric equilibrium.

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[1] In our analysis we have not considered the case where the rental rate "r" is equal to the wage rate "w". If r = w, then under simultaneous input choice the firms are indifferent between using labor and capital. Thus, corresponding to a unique equilibrium in the output space, there will be infinitely many solutions in the input space. However, under sequential input choice, the outcomes differ in two ways. First, there is an advantage of moving first, and hence, both the firms will be producing in the first stage using only capital. Neither firm will use the second stage input. Secondly, in this case, it can be shown that any outputs between (and including) the firms Stackelberg outcomes are sustainable as subgame perfect Nash equilibrium. Details can be found in Saloner (1987).

Since the profit of a Stackelberg follower is less than that of a leader, it is difficult to justify why a particular firm will play the role of a follower. Thus, a symmetric mixed strategy equilibrium seems more reasonable (if it exists) than an asymmetric pure strategy one. Therefore, now we extend our analysis to mixed strategies and focus on a symmetric mixed strategy equilibrium.

The existence of a symmetric equilibrium is known from the result that every symmetric game has a symmetric equilibrium.[2] It follows from Proposition 2.3 that, if $r \in (w, 0.03a + 0.97w)$, then there are only two asymmetric equilibria in pure strategies and, thus, there must be a symmetric equilibrium in mixed strategies.

However, solving for the mixed strategies becomes difficult, since the strategies available to each player are a continuum. Nevertheless, for each level of (K_1, K_2), a unique pure strategy solution for (L_1, L_2) follows and since labor decisions are taken after the realization of capital levels, mixed strategies can be eliminated during the labor choice. Therefore, the firms mix only over the levels of capital and in the second stage, after the realization of capital, a unique nonstochastic solution in the labor space follows accordingly. Also, from the shape of the profit functions we argue that the firms mix only over two different levels of capital, one being the Cournot level of capital (0) and the other being close to the Stackelberg leader's level (Q^L). In the following proposition, we find out the levels of capital and appropriate probabilities over these two points, which sustain this as a symmetric mixed strategy equilibrium.

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[2] This result can be found in Dasgupta and Maskin (1986). The existence of a symmetric mixed strategy equilibrium can also be argued from the result, which says that the total number of equilibria (including those in mixed strategies) in a game, cannot be even.

Proposition 2.4

If the inputs are perfect substitutes and $r \in (w,\ 0.03a + 0.97w)$, then a symmetric mixed strategy equilibrium (in the capital space) for the sequential input choice game is given by

$$K_i = \begin{cases} 0 \text{ with probability } p \\ Q^* \text{ with probability } (1 - p), \ \forall\ i = 1,\ 2 \end{cases}$$

where p and Q^* are the solutions of the following two equations,

$$Q^* = \{(a - r) - p[(a - w)/2]\}$$

$$p[(a - w)^2/9b] + (1 - p)(a - w - bQ^*)^2/4b = Q^*[a - r - 2bQ^*]$$

$$- p[(a - w)/2 - (3/2)bQ^*] \text{ with } p \in (0,1).$$

Proof: Let firm i mix over two points 0 and $Q^* > (a - w)/3b$ with probabilities p and $(1 - p)$ respectively. We search for p and Q^* such that

$$K_i = \begin{cases} 0 \ \text{ with probability } p \\ Q^* \text{ with probability } (1 - p) \end{cases}$$

is the best reply for firm j. From Figure 2.1 below it follows that the best reply to

$$K_i = \begin{cases} 0 \text{ with probability } \ p \\ Q^* \ \text{ with probability } (1 - p) \end{cases}$$

is either to choose $K_j = 0$ or to choose $K_j \geq (a - w)/3b$

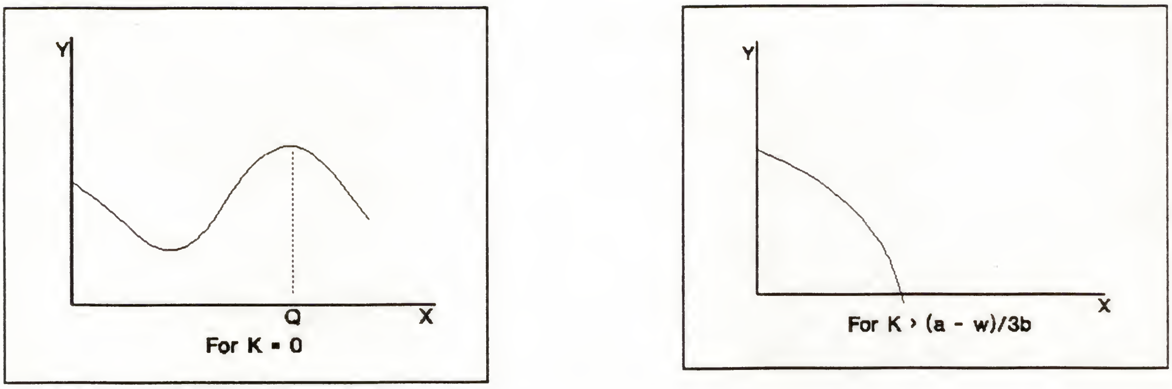

Figure 2.1

Shapes of profit functions for firm j, for given K_i
The X axis denotes K_j and the Y axis denotes π_j

Step 1 To find optimal $K_j \geq (a - w)/3b$

Suppose firm chooses $K_j = Q_j \geq (a - w)/3b$. Then $E(\pi_j \mid K_j = Q_j) = p\pi_j(0, Q_j) + (1 - p)\pi_j(Q^*, Q_j)$. Note that $\pi_j(0, Q_j) = Q_j[(a + w)/2 - bQ_j/2] - rQ_j$ and $\pi_j(Q^*, Q_j) = Q_j[a - bQ^* - bQ_j] - rQ_j$.

Hence $E(\pi_j \mid K_j = Q_j) =$

$pQ_j[(-a + w)/2 + bQ_j/2 + bQ^*] + Q_j[a - r - bQ^* - bQ_j]$

First order condition for profit maximization implies

$\partial E(\pi_j)/\partial Q_j = 0 \quad \Rightarrow$

$$Q_j = \{(a - r - bQ^*) + p[(-a + w)/2 + bQ^*]\} \qquad (2.1)$$

Step 2 To get p such that $E(\pi_j \mid K_j = 0) = E(\pi_j \mid K_j = Q_j)$

$E(\pi_j \mid K_j = 0) = p\pi_j(0, 0) + (1 - p)\pi_j(Q_*, 0)$

$= p(a - w)^2/9 + (1 - p)(a - w - bQ^*)^2/4b$

Hence p must satisfy,

$$p[(a - w)^2/9b] + (1 - p)(a - w - bQ^*)^2/4b = Q^*[a - r - bQ^* - bQ_j] - p[(a - w)/2 - bQ_j/2 - bQ^*] \qquad (2.2)$$

Since we are looking for symmetric mixed strategy equilibrium, the third condition is, $Q_j = Q^*$ (2.3)

Substituting (2.3) in equation (2.1) and (2.2) we get Proposition 3.4 ▲

The above proposition gives a symmetric mixed strategy equilibrium in the first-stage input space. However, at the end of the first stage four possible realizations of (K_1, K_2) are possible and corresponding to each realization of (K_1, K_2), a unique nonstochastic (L_1, L_2) follows. Thus, in the output space, there are four possible solutions with different probabilities. These outcomes are summarized in the following proposition.

Proposition 2.5

If the inputs are perfect substitutes and $r \in (w, 0.03a + 0.97w)$, then corresponding to the strategy

$$K_i = \begin{cases} 0 & \text{w.p.} \quad p \\ Q^* & \text{w.p.} \quad (1 - p) \end{cases} \qquad i = 1, 2$$

the following four outcomes occur with respective probabilities.

$\{(K_i = 0,\ L_i = Q^c),\ (K_j = 0,\ L_j = Q^c)\}$ with probability p^2.

$\{(K_i = Q^*,\ L_i = 0),\ (K_j = 0,\ L_j = (a - w)/2b - Q^*/2)\}$ w.p $p(1 - p)$, $i \neq j = 1, 2$

$\{(K_i = Q^*,\ L_i = 0),\ (K_j = Q^*,\ L_j = 0\}$ w.p $(1 - p)^2$

where Q^c is the output of a Cournot player, under simultaneous input choice.

Proof: Referring to the proof of Proposition 2.3, note that if $K_i \geq (a - w)/3b$, then $L_i = 0\ \forall\ K_j$. Also, if $K_i = 0$ and $K_j \geq (a - w)/3b$, then $L_i = (a - w)/2b - K_j/2\ \forall\ i \neq j = 1, 2$. Since $Q^* \geq (a - w)/3b$, Proposition 2.5 follows. ▲

In the above proposition the outcome $\{(K_i = 0, L_i = Q^c), (K_j = 0, L_j = Q^c)\}$ becomes identical with that under simultaneous input choice. Under simultaneous input choice, the firms never choose the costly input capital. They produce the entire output using the cheaper input labor. Here, with probability p^2, both the firms behave as followers not using any capital in stage one and, thus, producing the entire output in the second stage using the cheaper input labor and, hence, the outcome becomes identical with that under simultaneous input choice. The outcomes $\{(K_i = Q^*, L_i = 0), (K_j = 0, L_j = (a - w)/2b - Q^*/2)\}$ $i \neq j = 1, 2$ are close to the "leader-follower" case where one firm produces only in the first stage and the other produces only in the second stage, serving the remaining market. In the last outcome $\{(K_i = Q^*, L_i = 0), (K_j = Q^*, L_j = 0\}$, both the firms try to behave as leaders and, in fact, produce the entire amount in stage one using the costly input capital.

The above two Propositions 2.4 and 2.5 give us the solution in terms of Q^* and p. However, solving analytically for Q^* and p is difficult, since the reduced form of p is a cubic expression. Instead, we use numerical computation to get the values for p and Q^*, for different values of the parameters a, b, r and w.

In Table 2.1 (see next page), we present some of the results for different values of the parameters. Interestingly, those results show a significantly high value for p (greater than 0.9) reflecting the fact that in equilibrium a firm chooses the Cournot level of capital (0) with fairly high probability. However, to restrict its rival from behaving as a leader, it chooses a large capital Q^* with a small probability. Therefore,

Table 2.1

Equilibrium Mixed Strategies Under Different Values of The Parameters

$a = 100, b = 1, w = 4, r = 6$
$K_i = \begin{cases} 0 & \text{w.p. } 0.92 \\ 43.06 & \text{w.p. } 0.08 \end{cases}$

$a = 100, b = 2, w = 4, r = 6$
$K_i = \begin{cases} 0 & \text{w.p. } 0.92 \\ 21.51 & \text{w.p. } 0.08 \end{cases}$

$a = 50, b = 0.5, w = 10, r = 11$
$K_i = \begin{cases} 0 & \text{w.p. } 0.96 \\ 36.82 & \text{w.p. } 0.04 \end{cases}$

$a = 50, b = 1, w = 10, r = 11$
$K_i = \begin{cases} 0 & \text{w.p. } 0.96 \\ 18.36 & \text{w.p. } 0.04 \end{cases}$

$a = 50, b = 2, w = 10, r = 11$
$K_i = \begin{cases} 0 & \text{w.p. } 0.95 \\ 9.12 & \text{w.p. } 0.05 \end{cases}$

there is a fairly high chance (p^2) that the outcomes under symmetric mixed strategy becomes identical with those under traditional simultaneous input choice model.

The case of perfect substitutes explicitly shows a basis for strategic behavior other than cost complementarity. This is essentially "first mover advantage." However, if the inputs are perfect complements, then to produce any amount of output a firm needs both the inputs in a certain proportion. Investing in the first stage, a firm can ensure only its maximum level of output and any advantage of moving first gets significantly reduced. The absence of both cost complementarity and first mover advantage rules out the scope for strategic behavior, and hence, in the case of perfect complements, the solutions obtained under sequential vis-a-vis simultaneous input choice games are identical.

Proposition 2.6

If the inputs are perfect complements to each other, then the solutions to the sequential and simultaneous input choice games are identical.

Proof: First note that if the inputs are chosen simultaneously, then a unique pure strategy equilibrium is given by

$Q_i = K_i = L_i = \{a - (w + r)\}/3b \quad \forall\ i = 1, 2$

Next turn attention to the sequential input choice case. Here, $Q = \min [K, L]$ and therefore, given K (in the first stage), second stage choice of L has to be $\leq$ K. Thus, the second stage reaction function of firm i can be given by

$L_i = K_i \qquad \text{if } K_i \leq (a - w)/2b - L_j/2$, and

$L_i = (a - w)/2b - L_j/2$ if $K_i > (a - w)/2b - L_j/2$

Equivalently,

$L_i = K_i$ if $L_j \leq (a - w)/b - 2K_i$, and

$L_i = (a - w)/2b - L_j/2$ if $L_j > (a - w)/b - 2K_i$

Now we will show that in equilibrium $K_i = L_i \; \forall \; i = 1, 2$

Note that L_i can be written as $L_i = \{(a - w)/2b - L_j/2, K_i\}$

Suppose, $L_i < K_i \; \forall \; i = 1, 2 \Rightarrow L_i = (a - w)/2b - L_j/2 \Rightarrow L_i = L_j = (a - w)/3b$

So, K_i needs to be $> (a - w)/3b$ and firm i can increase profit by reducing K_i. Suppose in equilibrium $L_i = K_i$ and $L_j < K_j$. Then firm j can increase profit by reducing K_j. Therefore, the only possibility left with $L_i = K_i \; \forall \; i = 1, 2$.

Next we will show that in equilibrium $K_i = L_i \leq (a - w)/3b \; \forall \; i = 1, 2$. Also, in equilibrium $\partial L_j/\partial K_i = 0$. Note that for $K_i = L_i \; \forall \; i = 1, 2$, we need $(a - w)/b - 2K_i \geq (a - w)/3b \; \forall \; i = 1, 2$

$\Rightarrow K_i \leq (a - w)/3b \; \forall \; i = 1, 2$

Now if $K_i \leq (a - w)/3b$, then from the reaction functions in (L_1, L_2) space, it follows that $L_i = K_i \; \forall \; K_j \Rightarrow \partial L_i/\partial K_j = 0$.

We have shown that at equilibrium $L_i = K_i \; \forall \; i = 1, 2$, $K_i \leq (a - w)/3b \; \forall \; i = 1, 2$. Also, if $K_i \leq (a - w)/3b$, then $\partial L_i/\partial K_j = 0 \; \forall \; K_j$. These imply, at equilibrium, $L_i = K_i = Q_i \leq (a - w)/3b \; \forall \; i = 1, 2$.

Start from $K_i \leq (a - w)/3b$, and find out what is the best reply for firm j. Since $K_i \leq (a - w)/3b$, $L_i = K_i = Q_i \leq (a - w)/3b$. Also, $\partial L_i/\partial K_j = 0 \; \forall \; K_j$ So, for firm j it is optimal to choose efficiently. Thus firm j chooses $L_j = K_j = Q_j$. Therefore, firm j choose Q_j to maximize π_j. From the first order condition get, $Q_j = [a - (w + r)]/2b - Q_i/2$. (2.4)

Also, given $L_j = K_j = Q_j \leq (a - w)/3b$ the reaction function for firm i is

$$Q_j = [a - (w + r)]/2b - Q_i/2 \qquad (2.5)$$

Equation (2.4) and (2.5) imply a unique solution $Q_i = Q_j = [a - (r + w)/3b]$ in the range Q_i, $Q_j \leq (a - w)/3b$. However, it is already shown that if $Q_i > (a - w)/3b$ for at least one i, then it cannot be an equilibrium. Thus $Q_i = Q_j = [a - (r + w)/3b]$ is the unique solution for the sequential game which is identical to the solution of the simultaneous game. ▲

In the case of perfect substitutes, depending on factor prices, the sequential game does not have a symmetric solution (in pure strategies). However, in the case of imperfect substitute, a symmetric solution in pure strategies can exist. In fact, the following proposition shows that for a Cobb-Douglas technology of the form Q = KL, if a symmetric pure strategy equilibrium exists in the simultaneous game, then it does for the sequential game as well.

Proposition 2.7

> Given that the inputs are imperfect substitutes (Q = KL), if a symmetric pure strategy equilibrium exists in the simultaneous game, then it also does for the sequential game.

Proof : First note that for the sequential game, if $K_i = K_j = K$ is such that $4awK - 4w^2 - arbK^3 = 0$ and $K \geq 2w/a$, then (K, K) is an equilibrium pair in the capital space. This follows from the fact that for given $K_j \geq 2w/a$, optimal K_i must satisfy $\partial\pi_i/\partial K_i = 0$ (first order condition). This implies, $4awK_iK_j + 4w^2K_i - 8w^2K_j = 9rbK_i^3K_j$. Also, to satisfy second order conditions, we need $K_i \geq 3wK_j/(aK_j + w)$. If $K_i = K_j = K$ is such that $4awK - 4w^2 - arbK^3 = 0$, then K_i, K_j satisfy the first order conditions.

Now for (K_i, K_j) to be an equilibrium, we also need $K_i \geq 2w/a$ and $K_i \geq 3wK_j/(aK_j + w)$ $\forall$ $i \neq j = 1, 2$. Note that if $K_i = K_j = K$, then $K_i \geq 3wK_j/(aK_j + w) \iff K_i \geq 2w/a$. Hence, if (K_i, K_j) is such that, $K_i = K_j = K$ satisfies $4awK - 4w^2 - arbK^3 = 0$ and $K \geq 2w/a$, then (K_i, K_j) is an equilibrium in the capital space. Now note that if $a^3 > 18rbw$, then there exists $K \geq 2w/b$ such that $4awK - 4w^2 - arbK^3 = 0$. Therefore, if $a^3 > 18rbw$, the sequential game has a symmetric pure strategy equilibrium.

Next we will show that a necessary condition for the simultaneous game to have symmetric equilibrium is $a^3 \geq 20.25rbw$. Note that for the simultaneous game the reaction functions are given by, $a - 2bQ_i - bQ_j - \sqrt{(rw/Q_i)} = 0$. Now for a symmetric solution we need $Q_i = Q_j = Q$. Therefore, a symmetric solution does not exist unless $a - 3bQ - \sqrt{(rw/Q)} = 0$ has a solution. It can be checked that if $a^3 < 20.25rbw$, then $a - 2bQ_i - bQ_j - \sqrt{(rw/Q_i)} = 0$ does not have a positive solution.

Thus, a necessary condition for the existence of symmetric pure strategy equilibrium is $a^3 \geq 20.25rbw$, which is sufficient for existence of a symmetric solution in the sequential game. ▲

Section 2.4: Summary and Conclusions

We have considered a symmetric duopoly model under sequential input choice. In contrast to the standard duopoly model, a sequential choice of inputs provides scope for strategic behavior during input choice. We have identified two possible bases for strategic behavior: "cost complementarity" and "first mover advantage." In the case of perfect substitutes, "first mover advantage", while in the case of imperfect substitutes, ″cost complementarity″, provide the basis for strategic behavior. In the case of perfect complements neither ″cost complementarity″ nor ″first mover advantage″ are present and, hence, the results obtained under sequential vis-a-vis simultaneous games are identical.

We have also shown that in the case of ″first mover advantage,″ depending on the factor prices, the sequential game does not have any symmetric solution (in pure strategies), even with two identical firms. However, in the case of ″cost complementarity,″ a symmetric solution in pure strategies can exist. With a Cobb-Douglas technology of the form $Q = KL$, it is shown that, if a symmetric pure strategy equilibrium exists in the simultaneous game, then it does so for the sequential game.

We have obtained that in the case of perfect substitutes the ″first mover advantage″ is the basis for strategic behavior. However, this ″first mover advantage″ remains valid for any additively separable production function, or more generally, whenever the isoquants touch the axes. Thus, for a general kind of production function, where the isoquants are convex but touch the axes, ″cost complementarity″ and ″first mover advantage″ jointly provide the basis for strategic behavior. Since ″first mover advantage″ is associated with asymmetric ″leader-follower″ equilibria, while a symmetric pure strategy equilibrium is likely to exist in the case of ″cost complementarity,″ it is possible that, for a general neo-classical technology, the two different bases for strategic behavior might interact in such a way that a symmetric solution does not exist even in a symmetric model.

CHAPTER 3
COURNOT DUOPOLY WITH TWO PRODUCTION PERIODS AND COST DIFFERENTIALS

Section 3.1: Introduction

The standard Cournot oligopoly model is a one shot game. Saloner (1987) modifies the Cournot model by allowing two production periods before the market clears. In the first period the firms simultaneously decide initial production levels. These outputs become common knowledge and then the firms choose additional second period output simultaneously. The market clears only once, after the second period. Saloner shows that, in this case, any outcome on the outer envelope of the reaction functions between and including the firms' smallest Stackelberg outcomes is sustainable as a subgame perfect Nash equilibrium (SPNE). Saloner's analysis however, assumes identical costs of production over time.

In this chapter, we generalize Saloner's analysis by allowing cost differences across periods. In reality, the production costs seem likely to vary across periods. Even if the nominal production costs remain the same, allowing for discounting will make the real costs vary over time. When production costs are allowed to vary over time, it turns out that Saloner's results are "knife-edge" in the sense that the continuum of equilibria vanishes for any cost differential across periods. When production costs vary over time, depending on the cost differential, three kinds of SPNE (in pure strategies) are possible. If production is cheaper in the first period, then there is a unique SPNE at which both firms produce only in the first period, producing their single period Cournot-Nash quantities. If production cost is slightly higher in the first period, then there are two SPNE. At each, one firm behaves as a leader producing only in the first period, while the other behaves as a follower and produces only in the second period. However, if production cost is significantly higher in the first period, then again, there is a unique SPNE, but now the firms produce only in the second period. Thus, this analysis also addresses the question of endogenous timing in duopoly games. Whether duopolists play a simultaneous or sequential move game is determined endogenously in this model, rather than being assigned exogenously.

Section 3.2: The Model

We consider a symmetric duopoly model with two production periods, where the market clears at the end of the second period. In the first period, firms x and y simultaneously produce outputs q_1^x and q_1^y, respectively. These outputs become common knowledge and, in the second production period, the firms simultaneously choose nonnegative outputs q_2^x and q_2^y. After the second period, price is determined from the inverse demand function $P(q_1^x + q_2^x + q_1^y + q_2^y)$. The firms choose outputs to maximize profits. We assume that within a period the firms have the same constant marginal cost of production. However, production costs vary across periods. Let c_j be the marginal cost of production in period j, j = 1, 2.

For firm i, define the single-period reaction function[1]

$R^i(q^k|c) = \arg_z\max \; z\,(P(z + q^k) - c), \quad i \neq k = x, y$

We assume that these reaction functions are "well-behaved."[2] Let $(N^x(c_j), N^y(c_j))$ be the unique single-period Cournot-Nash equilibrium given the marginal cost c_j, $j = 1, 2$

A strategy for player i specifies an output for period one and an output for period two. Second period output is a function of the observed (q_1^x, q_1^y). We denote the strategy for firm i by $s^i = (s^i_1, s^i_2(q_1^x, q_1^y))$, $i = x, y$

For simplicity, we assume a unique Stackelberg point exists for each firm.[3] Denote these by $S^i(c_j) = \arg_z\max \; z\,[P(z + R^k(z|c_j)) - c_j]$, $i \neq k = x, y$, $j = 1, 2$

Section 3.3: Results and Discussion

The following lemma is used in proving our propositions. Saloner proved this lemma for $c_1 = c_2$. The result, however, remains valid for $c_1 \neq c_2$.

---

[1] By "single-period reaction function", we mean the reaction function corresponding to a standard Cournot model, which has only a single production period.

[2] By "well-behaved", we mean $0 > [\partial R^i(q^k|c_j) / \partial q^i] \geq -1$ and $[\partial R^i(q^k|c_j) / \partial c^j] < 0$. The first condition ensures the existence of a unique single-period Cournot-Nash equilibrium. The second condition is used to prove Proposition 3.2.
A set of sufficient condition for R^i functions to be "well-behaved" is. $P(q^i + q^k)$ is strictly positive on some bounded interval $(0, Q)$, on which it is twice continuously differentiable, strictly decreasing and concave. $P(q^i + q^k) = 0$ for $q^i + q^k \geq Q$.

[3] A downward sloping linear demand will be sufficient for the existence of a unique Stackelberg point.

Lemma 3.1

Given (q_1^x, q_1^y), the second-period equilibrium $s_2^i(q_1^x, q_1^y)$, $i = x, y$, is,

$$
\begin{aligned}
s_2^i(q_1^x, q_1^y) &= 0 && \text{if } q_1^i \geq R^i(q_1^k|c_2),\ q_1^k \geq R^k(q_1^i|c_2) \\
&= N^i(c_2) - q_1^i && \text{if } q_1^i \leq N^i(c_2),\ q_1^k \leq N^k(c_2) \\
&= 0 && \text{if } q_1^i \geq N^i(c_2),\ q_1^k \leq R^k(q^i|c_2) \\
&= R^i(q_1^k|c_2) - q_1^i && \text{if } q_1^i \leq N^i(c_2),\ q_1^k \geq N^k(c_2), \\
& && \quad q_1^i \leq R^i(q_1^k|c_2)
\end{aligned}
$$

This lemma says if (q_1^x, q_1^y) lies on or outside the outer envelope of $(R^x(c_2), R^y(c_2))$, then neither firm produces in the second period. If both firms produce less than $(N^x(c_2), N^y(c_2))$ (i.e., second period Cournot outputs) in the first period, then each produces up to its second period Cournot level. If one exceeds its second period Cournot output and the other does not, the latter produces in the second period its best response to the former's first period output and the former does not produce.

We next briefly review Saloner's result. It will be useful to understand why the production cost differential plays a crucial role in determining the equilibria in this model.

Proposition 3.1 (due to Saloner)

If $c_1 = c_2$, then any outcome on the outer envelope of the reaction functions between (and including) the firms' Stackelberg outcomes is sustainable as a SPNE.

Proof: See Saloner (1987). ▲

The intuition behind this result is as follows. Let (x^*, y^*) be the total output produced by firms x and y, respectively. Assume that (x^*, y^*) is on the outer envelope formed by $R^x(q^y)$ and $R^y(q^x)$, and between (and including) S^x and S^y.[4] Without loss of generality let (x^*, y^*) be on $R^y(q^x)$. According to the above result this can be a SPNE. Note that, if there were only one production period then $(x^*, y^*) \neq (N^x, N^y)$ would not be sustainable as a Nash equilibrium since when firm y produces y^*, firm x would do better to produce less than x^*. In the case of two periods, however, $\{(s_1^x = x^*,\ s_2^x = 0), (s_1^y = y^*,\ s_2^y = 0)\}$ sustains (x^*, y^*) as a SPNE. Here, unlike the one period case, firm x cannot gain by choosing $s_1^x < x^*$. If it does, then from Lemma 3.1, firm y would produce $s_2^y = R^y(q^x) - y^* > 0$ in the second period. This would lower firm x's profits below what it gets at (x^*, y^*), and thus (x^*, y^*) is sustained as a SPNE.

Next consider production costs differing across periods. Proposition 3.2 gives the result for $c_1 < c_2$.

Proposition 3.2

If $c_1 < c_2$, then the unique SPNE of this two period Cournot model is $\{(s_1^x = N^x(c_1),\ s_2^x = 0),\ (s_1^y = N^y(c_1),\ s_2^y = 0)\}$.

Proof: Since $c_1 < c_2$, $R^i(q^k|c_2)$ lies inside $R^i(q^k|c_1)$ $i \neq k = x,y$. Now consider the outer envelope formed by $R^y(q^x|c_1)$ and $R^x(q^y|c_1)$. Let (x^*, y^*) denote any vector of total outputs that can be sustained as a SPNE. (x^*, y^*) cannot lie either outside or inside this outer envelope. If it lies outside, then both the firms can gain by producing less. If it lies inside, then at least one of them can gain by producing more in the first period. Hence, we can restrict our attention to points on this outer envelope. Now, if $x^* > S^x(c_1)$ (or $y^* > S^y(c_1)$), then firm x (or y) can gain by producing less. This further restricts the candidate points for SPNE to the outer envelope between (and including) $S^x(c_1)$ and $S^y(c_1)$. Suppose (x^*, y^*) lies on this locus. Since $c_1 < c_2$, if (x^*, y^*) is sustainable as a SPNE, it has to be $\{(s_1^x = x^*, s_2 = 0), (s_1^y = y^*, s_2 = 0)\}$. Note that, when $c_1 = c_2$, these strategies sustain all the outcomes on this region as SPNE. However, if $c_1 < c_2$ and $(x^*, y^*) \neq (N^x(c_1), N^y(c_1))$ lies on $R^y(q^x|c_1)$, then firm x can do better by producing slightly less than x^*. Since $c_1 < c_2$, $R^y(q^x|c_2) < R^y(q^x|c_1)$. Thus, $\exists$ an $\varepsilon > 0$ such that if $s_1^x = x^* - \varepsilon$, s_2^y is still zero. Thus firm x can gain by choosing $s_1^x < x^*$, and (x^*, y^*) cannot be sustained as a SPNE. Similarly if $(x^*, y^*) \neq (N^x(c_1), N^y(c_1))$ lies on $R^x(q^y|c_1)$, then firm y can gain by choosing $s_1^y < y^*$. The only possibility left is $(x^*, y^*) = (N^x(c_1), N^y(c_1))$, i.e., the single-period Cournot-Nash outcome given the first period cost c_1. Clearly $\{(s_1^x = N^x(c_1), s_2^x = 0), (s_1^y = N^y(c_1), s_2^y = 0)\}$ sustains $(N^x(c_1), N^y(c_1))$ as a SPNE. ▲

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[4] Since $c_1 = c_2$, $R^i(q^k|c_1) = R^i(q^k|c_2)$ and $S^i(c_1) = S^i(c_2)$. This allows us to omit the cost parameters.

Next, Proposition 3.3 characterizes the SPNE when $c_1 > c_2$. Lemma 3.2 and 3.3 are used to prove Proposition 3.3.

Lemma 3.2

If $q_1^i > N^i(c_2)$, then the optimal $s_1^k = 0$, and

if $q_1^i \in [0, N^i(c_2)]$, then the optimal $s_1^k \notin (0, N^k(c_2)]$. $i \neq k = x,y$

Proof: If $q_1^i > N^i(c_2)$ then $s_2^i = 0$ for any s_1^k. (This follows from Lemma 3.1). Then since $c_2 < c_1$, it is best for firm k to choose $s_1^k = 0$ and produce its best response in the second period.

If $q_1^i \in [0, N^i(c_2)]$ and the optimal $s_1^k = q_1^k \in (0, N^k(c_2)]$, then from the second period's equilibrium strategies (Lemma 3.1) we have that, $s_2^i = N^i(c_2) - q_1^i$ and $s_2^k = N^k(c_2) - q_1^k$. Clearly firm k can do better by choosing $s_1^k = 0$, because $c_2 < c_1$. ▲

Lemma 3.3

The feasible strategies which sustain a SPNE must satisfy either one of the following.

$\{(s_1^x = 0,\ s_2^x = N^x(c_2)),\ (s_1^y = 0,\ s_2^y = N^y(c_2))\}$

$\{(s_1^x = q_1^x > N^x(c_2),\ s_2^x = 0),\ (s_1^y = 0,\ s_2^y = R^y(q_1^x | c_2))\}$

$\{(s_1^x = 0,\ s_2^x = R^x(q_1^y | c_2)),\ (s_1^y = q_1^y > N^y(c_2),\ s_2^y = 0)\}$

Proof: Follows from Lemma 3.2 ▲

Proposition 3.3

If $c_1 > c_2$ then $\exists$ a $c_1^* > c_2$ such that,

(1) If $c_1 > c_1^*$, then the unique SPNE of this two period Cournot model is $\{(s_1^x = 0,\ s_2^x = N^x(c_2)),\ (s_1^y = 0,\ s_2^y = N^y(c_2))\}$

(2) If $c_1 < c_1^*$, then there are two SPNE, as given by

$\{(s_1^x = x^L,\ s_2^x = 0),\ (s_1^y = 0,\ s_2^y = R^y(x^L | c_2))\}$

$\{(s_1^x = 0,\ s_2^x = R^x(y^L | c_2)),\ (s_1^y = y^L,\ s_2^y = 0)\}$

(3) If $c_1 = c_1^*$, then there are three SPNE, as given by the strategies in (1) and (2).

Proof: Suppose $s_1^y = 0$. Then for firm x, the optimal s_1^x is either 0 or greater than $N^x(c_2)$. (This follows from Lemma 3.2.) If $s_1^x = 0$, then in the second period $s_2^x = N^x(c_2)$ and $s_2^y = N^y(c_2)$. In this case, firm x's profit is $[\pi^C(x)] = N^x(c_2)\ [P(N^x(c_2) + N^y(c_2)) - c_2]$. However, if $s_1^x > N^x(c_2)$ then the optimal $s_1^x = \arg\max_z z\ [P(z + R^y(z|c_2)) - c_1]$. Let us assume $x^L(c_1)$ uniquely solves this problem, so that $s_1^x = x^L(c_1)$.[5]

Then firm x's profit $[\pi^L(x)] = x^L\ [P(x^L + R^y(x^L|c_2)) - c_1]$.

Hence, $s_1^x = 0$ or $x^L(c_1)$ accordingly $[\pi^C(x)] {}^{>}_{<} [\pi^L(x)] \Leftrightarrow c_1 {}^{>}_{<} c_1^*$ where c_1^* is such that $[\pi^C(x)] = [\pi^L(x)]$. That is,

$N^x(c_2)\ [P(N^x(c_2) + N^y(c_2)) - c_2] = x^L(c_1^*)\ [P(x^L(c_1^*) + R^y(x^L(c_1^*)|c_2) - c_1^*]$

Similarly, for $s_1^x = 0$, $s_1^y = 0$ or y^L according as $c_1 {}^{>}_{<} c_1^*$.

Hence, if $c_1 > c_1^*$, then $\{(s_1^x = 0,\ s_2^x = N^x(c_2)),\ (s_1^y = 0,\ s_2^y = N^y(c_2)\}$ is the unique SPNE, whereas if $c_1 < c_1^*$ then both $\{(s_1^x = x^L,\ s_2^x = 0),\ (s_1^y = 0,\ s_2^y = R^y(x^L|c_2))\}$ and $\{(s_1^x = 0,\ s_2^x = R^x(y^L|c_2)),\ (s_1^y = y^L,\ s_2^y = 0)\}$ are SPNE.

Also, for $c_1 = c_1^*$, $[\pi^C(x)] = [\pi^L(x)]$. Thus all the strategies in Lemma 3.3 generate SPNE. ▲

The intuition behind this proposition is as follows. By producing in the first period, a firm gets the opportunity to behave as a leader. However, with $c_1 > c_2$, production is costlier in period one. So a firm will be a leader only if the benefit of being a leader dominates the loss from the use of a costlier technology. As a result, if c_1 is not "much" higher than c_2, we get two equilibria, at each of which one of the firms behaves as a leader and produces the entire output only in the first period. The other behaves as a follower and produces only in the second period. For $c_1 > c_1^*$, the gain from behaving as a leader no longer dominates the loss due to the use of costlier technology, thus both firms produce their best response outputs in the second period and produce their Cournot-Nash quantity (relative to the second period's cost).

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[5] A downward sloping linear demand will be sufficient for uniqueness.

Section 3.4: Summary and Conclusions

One important feature distinguishing different duopoly models is whether firms move simultaneously or whether they move sequentially. Much of the traditional duopoly analysis has treated this feature as exogenously given. Recently, recognition has been given to the fact that whether duopolists play a simultaneous or sequential move game should not be exogenous but should be determined endogenously from the model.[6] This duopoly model with two production periods addresses the question of endogenous timing in duopoly games. For $c_1 < c_2$ and $c_1 > c_1^* > c_2$, the model generates the Cournot-Nash equilibria as the unique SPNE of the game. However, in the former case the firms produce their entire output simultaneously in the first period, whereas in the latter case they produce only in the second period. Stackelberg outcomes are generated endogenously for $c_2 < c_1 < c_1^*$. In this case, one of the firms behaves as a leader and produces its entire output in the first period. The other behaves as a follower and produces its best response in the second period. However, for $c_1 = c_2$ and $c_1 = c_1^*$, we encounter multiple SPNE, where both Cournot-Nash and Stackelberg outcomes can result endogenously from the model.

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[6] See Boyer and Moreaux (1987), Dowrick (1986), Gal-Or (1985), Hamilton and Slutsky (1989), Mailath (1988), Reinganum (1985)

CHAPTER 4
DUOPOLISTIC COMPETITION IN TWO-PART TARIFFS

Section 4.1: Introduction

In traditional price competition, the pricing policies studied are mostly limited to uniform prices. However, in reality, we often encounter price competition with nonlinear price schedules, in particular in two-part tariffs, where in addition to a per unit price, the consumer must pay a fixed fee in order to buy any positive amount. For example, often, in the case of video rentals a consumer first needs to be a member. To rent movies, a membership fee is paid, in addition to a per unit price for movie rental. Car rentals are another example where the consumer pays a daily fee in addition to a mileage charge. The purpose of this paper is to extend traditional price competition by allowing the firms to compete in two-part price.

Two-part pricing has been extensively discussed in the literature, but mostly in the context of monopoly. In the case of a regulated monopoly, two-part pricing is used to achieve efficiency, whereas, for a profit maximizing monopoly, it is a device to extract surplus from the consumers. However, in the context of a competitive market structure, it has drawn very limited attention. Calem and Spulber (1984) discuss an oligopoly without entry in which firms produce differentiated products. In the case of homogeneous products, two-part pricing has been considered in the context of shared facilities, where congestion plays a crucial role.[1] However, we often observe competitive two-part pricing in the absence of congestion. For example, car rental is a pure private good and is not subject to congestion as long as total demand does not exceed maximum capacity. The existing literature on competitive two-part pricing has not paid attention to this case, where the firms produce identical products and consumption is free of congestion.

In this chapter, we take up the case of a homogeneous product with no congestion or capacity constraint. We consider two identical firms and infinitely many consumers with varying tastes. Robustness of some well known results (derived under uniform pricing) is examined. Under uniform pricing, two firms are sufficient to bring down the equilibrium price to marginal cost, and thus ensure zero profit for each. This result is invariant to the assumptions about timing of price setting, that is, whether or not we consider a sequential or simultaneous price setting model, the outcomes remain the same. However, it is shown that under two-part tariffs with sequential price setting, the firms can coexist in equilibrium with positive profits by serving different segments of the market. The price leader serves the low demanders, whereas, the price follower serves the high demanders. The question of entry is examined next. Under uniform pricing any positive entry cost can create a complete entry barrier. It is shown that, under sequential price setting two-part tariffs promote entry into the market, whether or not the entrant is a price leader or a price follower. In fact, when the incumbent is the price leader and the entrant is the follower, entry is facilitated even if only the incumbent has the option of charging a two-part tariff and the entrant does not. In this case, we also reexamine contestability results under two-part tariffs and show that one potential entrant with zero entry cost is no longer sufficient to ensure zero profit for the incumbent. Finally we analyze competition under two-part tariffs with simultaneous price setting. It is shown that, in this case, the outcomes under uniform pricing vis-a-vis two-part tariffs are the same.

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[1] The literature on shared facilities is called club theory and discussion can be found in Berglas (1981), Boadway (1980), Buchanan (1965), Scotchmer (1985).

This chapter is organized as follows. Section 4.2 describes the model. Sections 4.3 and 4.4 discuss after entry price competition and entry, without and with the option of two-part pricing, respectively. Section 4.5 examines the contestability doctrine under two-part pricing. Section 4.6 discusses competition with simultaneous price setting, and Section 4.7 concludes the chapter.

Section 4.2: The Basic Model

Our model consists of one incumbent monopolist, one potential entrant and infinitely many consumers. Both the incumbent and the entrant have identical technologies to produce a single homogeneous good q. Production of the good q can be done costlessly. However, there is a positive entry cost involved. This entry cost is sunk in the sense that, once paid it cannot be recovered. Each firm's price schedule has two components (F, p), where p is an unit price and $F \geq 0$ is a fixed fee which must be paid by the consumer to consume any positive amount. If the entrant decides to enter, then a sequential price competition between the two firms follows. One of the firms act as a price leader by announcing its price schedule first. The other observes the leader's price schedule and then reacts. However, at this point we do not specify who is the leader and who is the follower.

Consumers have varying tastes for the product. A consumer of type θ has demand of the form $p = \theta - q$, where p is price and θ is the taste parameter. We assume θ is uniformly distributed over an interval $[0, \theta^u]$. Trade among the consumers is not permitted. If a consumer of type θ buys from firm i with price schedule (F^i, p^i), she buys an amount $q(\theta) = \theta - p^i$, that is, the "income effect" is ignored. Given a price schedule (F, p) define consumer surplus for type θ as $CS(\theta|(F, p)) = 0.5(\theta - p)^2 - F$. Then the consumer buys from firm i if $CS(\theta|(F^i, p^i)) > CS(\theta|(F^j, p^j))$ and $CS(\theta|(F^i, p^i)) \geq 0$. If $CS(\theta|(F^i, p^i)) = CS(\theta|(F^j, p^j)) \geq 0$, then the consumer is indifferent between firm i and j.

We consider a two-stage game. In stage one, the entrant decides whether or not to enter. If it decides to enter, then, in the next stage, price competition between the existing incumbent and the entrant follows. We are looking for a subgame perfect Nash equilibrium. Thus, the entrant enters the market if and only if its post entry equilibrium profit is not less than the entry cost. We solve the model by backward induction. That is, first we consider the post entry price competition between the two firms. Then in the second stage, the entrant's after entry equilibrium profit is compared with the entry cost, to decide whether entry occurs or not.

Section 4.3: Competition and Entry with Uniform Price

Suppose both the price leader and the price follower are restricted to charge an uniform price. Since there is no production cost, as long as the leader's price $p^l > 0$, the follower's optimal response is to undercut it. Thus, the equilibrium price comes down to zero, which gives zero equilibrium profit for both the leader and the follower. Therefore, whether or not the entrant is a price leader or a follower, its post entry equilibrium profit is zero. Since a positive entry cost must be paid, the entrant's net payoff from entering the market is negative and hence, entry never occurs. Thus, in this case, the incumbent can charge its monopoly price and still block entry.

Section 4.4: Competition and Entry with Two-part Price

Now we consider the situation, where the firms can charge a two-part tariff. Under two-part pricing the consumer pays a fixed fee F and a unit price p. The price leader announces a fixed fee and unit price combination (F^l, p^l) first, and then the follower reacts by choosing a pair of fixed fee and unit price (F^f, p^f). Thus, it is always feasible for the follower to provide greater or equal amount of surplus (compared to the amount of surplus they would get from the leader) to the consumers. That is the follower can always undercut or match the leader. However, now the follower has another choice - it can accommodate the leader, that is, instead of providing a better (or as good as) deal to all the consumers, it can allow the leader to serve one segment of the market and itself serve the other. This is possible since the consumers differ in tastes. Roughly speaking, for the consumers with low θ (low demanders), a lower fixed fee and higher unit price is preferable to a higher fixed fee and lower unit price, whereas for the consumers high θ (high demanders) the converse is true. Thus, for given leader's (F^l, p^l), the follower can charge either $(F^f > F^l, p^f < p^l)$ or $(F^f < F^l, p^f > p^l)$. In the first case the consumers with high θ (high demanders) get more surplus from the follower, whereas, for the consumers with low θ (low demanders) the converse is true. In the second case $(F^f < F^l, p^f > p^l)$, the follower serves the low demanders, whereas, the leader serves the high demanders. In other words, now it is feasible for the follower to accommodate the leader by letting it serve the low (high) demanders while itself serving the high (low) demanders only.

However, the question is whether it is profitable for the follower to accommodate the leader. That is, whether the follower's accommodating profit is higher than its matching/undercutting profit. Since both the actions (accommodating and undercutting/matching) are feasible for the follower, its credible response depends on which action is more profitable.

In the next proposition we show that the follower does not accommodate the leader if the latter concentrates on the high demanders. That is, given the leader's (F^l, p^l), the follower's optimal choice can never be $(F^f < F^l, p^f > p^l)$ with the leader serving the high demanders.

Proposition 4.1

When the consumers differ in tastes, then in the price game, we can never have an equilibrium with the follower serving the low demanders and the leader serving the high demanders.

Proof: This proof is by contradiction. Suppose $\exists$ an equilibrium in the price game where the follower serves the low demanders and the leader serves the high demanders. Then $\exists\ \theta^* \in (0, \theta^u)$ such that the leader serves demanders with $\theta \geq \theta^*$ and the follower serves demanders with $\theta^l \leq \theta \leq \theta^*$, where $\theta^l = \min\{\theta \mid 0.5(\theta - P^f)^2 - F^f = 0\}$. Then the follower's profit is given by

$$\pi^f(\theta^*) = \int_{\theta^l}^{\theta^*} \{P^f(\theta - P^f) + F^f\}\, d\theta, \text{ where } \theta^* \text{ is such that}$$

$0.5(\theta^* - P^l)^2 - F^l = 0.5(\theta^* - P^f)^2 - F^f$, $[F^f < F^l\ ,\ P^f > P^l]$, (P^l, F^l) and (P^f, F^f) are chosen optimally by the leader and the follower, respectively. Let $\pi^f(\theta^* + \varepsilon)$ be the profit of the follower if it serves up to $(\theta^* + \varepsilon)$, that is, $\pi^f(\theta^* + \varepsilon) = \int_{\theta^l}^{\{\theta^* + \varepsilon\}}\{P^f(\theta - P^f) + F^f\}d\theta$, where

$0.5(\theta^* + \varepsilon - P^l)^2 - F^l = 0.5(\theta^* + \varepsilon - P^f)^2 - F^f$, $F^f < F^l$ and $P^f > P^l$.

We first find the optimal P^f and F^f if the follower serves up to $(\theta^* + \varepsilon)$. Substituting them back in the profit function we get the maximized $\pi^f(\theta^* + \varepsilon)$, say $\pi^{f*}(\theta^* + \varepsilon)$. Then we evaluate $\partial\pi^{f*}(\theta^* + \varepsilon)/\partial\varepsilon$ at $\varepsilon = 0$ and show that it is positive. This implies that the follower can do better by serving beyond θ^*, and thus we get a contradiction to the equation where the follower serves up to θ^*.

Step 1: To find the optimal (P^f, F^f)

$$\text{Maximize } \pi^f(\theta^* + \varepsilon) = \int_{\theta^l}^{\{\theta^* + \varepsilon\}}\{P^f(\theta - P^f) + F^f\}d\theta$$

subject to $0.5(\theta^* + \varepsilon - P^l)^2 - F^l = 0.5(\theta^* + \varepsilon - P^f)^2 - F^f$

Note that $\theta^l = P^f + \surd(2F^f)$ and the corresponding Lagrangian is given by

$$L = \int_{\theta^l}^{\{\theta^* + \varepsilon\}} \{P^f(\theta - P^f) + F^f\}d\theta - \lambda\{0.5(\theta^* + \varepsilon - P^l)^2 - F^l - 0.5(\theta^* + \varepsilon - P^f)^2 + F^f\}$$

First order conditions for profit maximization imply,

$$\partial L/\partial P^f = \int_{\theta^l}^{\{\theta^* + \varepsilon\}} (\theta - 2P^f)d\theta - \{P^f\sqrt{(2F^f)} + F^f\} - \lambda\{(\theta^* + \varepsilon - P^f)\} = 0 \qquad (4.1)$$

$$\partial L/\partial F^f = \int_{\theta^l}^{\{\theta^* + \varepsilon\}} d\theta - \{P^f\sqrt{(2F^f)} + F^f\}\{1/\sqrt{(2F^f)}\} - \lambda = 0 \qquad (4.2)$$

Solving the above two equations (4.1) and (4.2), we get the optimal P^f and F^f must satisfy $\theta^* + \varepsilon - P^f - 2\sqrt{(2F^f)} = 0$ (4.3)

Step 2: To evaluate $\partial\pi^{f*}(\theta^* + \varepsilon)/\partial\varepsilon$ at $\varepsilon = 0$

Using the envelope theorem

$\partial\pi^{f*}(\theta^* + \varepsilon)/\partial\varepsilon = \partial L/\partial\varepsilon$

$= \{P^f((\theta^* + \varepsilon - P^f) + F^f\} - \lambda\{(\theta^* + \varepsilon - P^l) - (\theta^* + \varepsilon - P^f)\}$

$= \{P^f((\theta^* + \varepsilon - P^f) + F^f\} - \lambda(P^f - P^l)$

From equation (4.2), $\lambda = (\theta^* + \varepsilon - P^f - \sqrt{(2F^f)}) - \{P^f\sqrt{(2F^f)} + F^f\}\{1/\sqrt{(2F^f)}\}$

$= \theta^* + \varepsilon - 2P^f - \sqrt{(2F^f)} - (\sqrt{(F^f)}/\sqrt{(2)})$

Substituting P^f from equation (4.3), get $\lambda = (5/\sqrt{(2)})\sqrt{(F^f)} - \theta^* - \varepsilon$

After simplifying we get, $\partial\pi^{f*}(\theta^* + \varepsilon)/\partial\varepsilon$ (at $\varepsilon = 0$)

$= 3F^f - (5/\sqrt{(2)})\sqrt{(F^f)}(\theta^* - P^l) + \theta^*(\theta^* - P^l)$

Step 3: To show $\partial\pi^{f*}(\theta^* + \varepsilon)/\partial\varepsilon$ (at $\varepsilon = 0$) is positive

Claim $F^f < (1/8)(\theta^* + \varepsilon - P^l)^2$

Proof: Note that $0.5(\theta^* + \varepsilon - P^l)^2 - F^l = 0.5(\theta^* + \varepsilon - P^f)^2 - F^f$

$\Rightarrow F^f = (1/3)\{0.5(\theta^* + \varepsilon - P^l)^2 - F^l\}$

Now $F^f < F^l \Rightarrow F^l > (1/8)(\theta^* + \varepsilon - P^l)^2 \Rightarrow F^f < (1/8)(\theta^* + \varepsilon - P^l)^2$

Call $g(F^f) = 3F^f - (5/\surd(2))\surd(F^f)(\theta^* - P^l) + \theta^*(\theta^* - P^l)$

Diagram 4.1 gives the shape of $g(F^f)$. The X axis denotes F^f and the Y axis denotes $g(F^f)$.

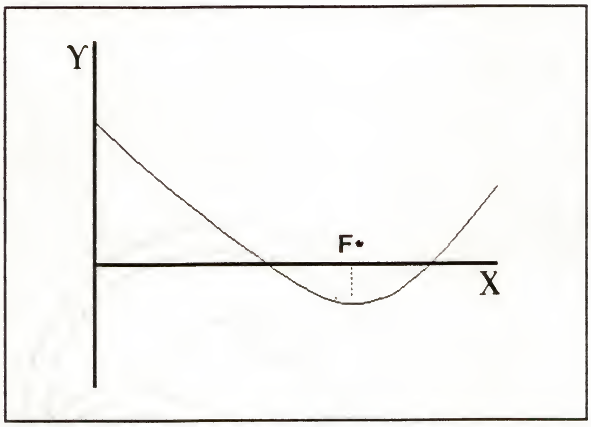

Figure 4.1

Let $g'(F*) = 0$. Then $(1/8)(\theta^* - P^l)^2 < F*$.

Also note that $g((1/8)(\theta^* - P^l)^2) > 0$.

$\Rightarrow g(F^f) > 0 \ \forall \ F^f < (1/8)(\theta^* - P^l)^2$.

Therefore $\partial\pi^{f*}(\theta^* + \varepsilon)/\partial\varepsilon$ (at $\varepsilon = 0$) > 0 and we have a contradiction. ▲

The intuition behind this result is as follows. If the leader concentrates on the high demanders, then to accommodate it the follower must confine itself to the low demanders. However, in this situation, undercutting is a better strategy, since the follower can capture the high demanders at the cost of some of the low demanders. Since the high demanders generate more revenue than the low demanders, the follower prefers undercutting to accommodating.

Since the follower does not let the leader serve the high demanders, the only hope left to the leader is to concentrate on the low demanders.

To explore this possibility, we consider two cases. In the first case, only the follower has the privilege of charging a two-part price; the leader is restricted to a uniform price. In case two, both the follower and the leader can charge two-part price schedules. The first case helps to understand the intuition behind our results with less analytical complications.

We first examine Case one. In this situation the leader can provide an attractive deal only to the low demanders (low θ), since they are the ones who prefer lower fixed fee and higher unit price to higher fixed fee and lower unit price. Now, to undercut the leader, the follower will have to match the offer for the low demanders. This implies that it will be restricted to a lower fixed fee from everybody. Whereas, by accommodating the leader, the follower serves only the high demanders and thus can charge a higher fixed fee F. Hence, by accommodating the leader, although the follower looses the low demanders, it can charge higher fixed fee from the high demanders. Therefore, it might be profitable for the follower to accommodate the price leader. The following results show that, in fact, the follower accommodates the leader, if the latter restricts itself to the low demanders.

Lemma 4.1

If the leader's price $p^l \in \{(1/3)\theta^u, (3/5)\theta^u\}$ then the follower chooses ($F^f > 0$, $p^f < p^l$) such that there is a $\theta^* \in (0, \theta^u)$, where all consumers with $p^l < \theta < \theta^*$ buy from the leader while consumers with $\theta > \theta^*$ buy from the follower.

Proof: Given the leader's price P^l only those consumers with $\theta > P^l$ get a positive surplus from the leader. However, it is feasible for the follower to provide an equal/greater amount of surplus to all these consumers. To do so, he must provide the consumer at $\theta = P^l$ an equal amount of surplus (compared to what she would get from the leader). Then everybody with $\theta > P^l$ gets a higher surplus from the follower. We will show that the follower can do better by not serving the consumer at $\theta = P^l$ and thus $\forall\ \theta^* > P^l$ such that the follower serves from θ^* to θ^u and the leader serves from P^l to θ^*.

Suppose the follower serves from $(P^l + \varepsilon)$ to θ^u. Then we find out what is the optimal P^f and F^f. Substitute them back into the profit function $\pi^f(P^l + \varepsilon)$. We evaluate $\partial\pi^f(P^l + \varepsilon)/\partial\varepsilon$ at $\varepsilon = 0$ and show that it is positive. This shows that the follower can do better by not serving the consumer at $\theta = P^l$.

Step 1: To find the optimal P^f and F^f.

$$\text{Maximize } \pi^f(P^l + \varepsilon) = \int_{\{p^l + \varepsilon\}}^{\theta^u} \{P^f(\theta - P^f) + F^f\} d\theta$$

$$\text{subject to } \ 0.5(P^l + \varepsilon - P^f)^2 - F^f = 0.5(P^l + \varepsilon - P^l)^2 = \varepsilon^2/2$$

The corresponding Lagrangian is given by

$$L = \int_{\{p^l + \varepsilon\}}^{\theta^u} \{P^f(\theta - P^f) + F^f\} d\theta - \lambda\{.5(P^l + \varepsilon - P^f)^2 - F^f - \varepsilon^2/2\}$$

First order conditions for profit maximization imply,

$$\int_{\{p^l + \varepsilon\}}^{\theta^u} (\theta - 2P^f) d\theta + \lambda(P^l + \varepsilon - P^f) = 0 \tag{4.4}$$

$$\text{and } \int_{\{p^l + \varepsilon\}}^{\theta^u} d\theta + \lambda = 0 \qquad (4.5)$$

Solving equation (4.4) and (4.5) we get,

$P^f = (\theta^u - P^l - \varepsilon)/2$ and $F^f = .5(P^l + \varepsilon - P^f) - \varepsilon^2/2$

Step 2: To show $\partial\pi^{f*}(P^l + \varepsilon)/\partial\varepsilon$ (at $\varepsilon = 0$) is positive

$\partial\pi^{f*}(P^l + \varepsilon)/\partial\varepsilon = \partial L/\partial\varepsilon = -\{P^f(P^l + \varepsilon - P^f) + F^f\} - \lambda(P^l - P^f)$

After simplifying get $\partial\pi^{f*}(P^l + \varepsilon)/\partial\varepsilon$ (at $\varepsilon = 0$) $= (1/8)(3P^l - \theta^u)(3\theta^u - 5P^l)$

Since $(1/3)\theta^u < P^l < (3/5)\theta^u$, this is positive. ▲

Proposition 4.2 follows from Lemma 4.1.

Proposition 4.2

When the leader is restricted to an uniform price, there is a feasible set of prices for the leader such that the follower prefers to accommodate rather than undercut the leader, and as a result, the leader serves the low demanders and the follower serves the high demanders.

From Proposition 4.2 it directly follows that the leader's market share and hence profit is positive. From Lemma 4.1 it follows that the market is segmented into low and high demanders, and the follower serves the high demanders allowing the leader to serve the low demanders. Thus both of them can avoid head to head price competition by serving different groups of consumers and coexist with positive profits.

Compared with the uniform pricing case (Section 4.3), it can be seen that although the leader is restricted to a uniform price, the follower's privilege of charging a two part price also helps the leader. When the follower too is restricted to charging an uniform price, it is forced to compete head to head with the leader and as a result of this price war both the firms' profits come down to zero. However, the follower's privilege of charging two-part price provides room to avoid head to head price competition. As a result, the follower accommodates the leader and the latter makes positive profit by serving the low demanders.

Now we come to Case two. The question is, can the leader charge an entry fee $F^l > 0$ and still get an accommodating response from the follower ? Qualitatively, the idea behind Proposition 4.2 goes through as long as F^l is small and p^l is high enough to confine the leader to the low demanders. The next proposition shows that even when both of them are charging two part price, the leader can still get an accommodating response if it restricts itself to the low demanders.

Proposition 4.3

> When both the leader and the follower can charge two-part tariffs, there is a feasible set of (F^l, p^l) for the leader such that the follower prefers to accommodate rather than undercut the leader, and as a result, the leader serves the low demanders and the follower serves the high demanders.

Proof: This proof is similar to that of Lemma 4.1. Now the price leader charges $F^l > 0$, in addition to $P^l > 0$. Let the consumer at $\theta = \theta^l$ gets zero surplus from the leader, that is, $0.5(\theta^l - P^l)^2 - F^l = 0$.

Evaluate $\partial\pi^{f*}(\theta^* + \varepsilon)/\partial\varepsilon$ at $\varepsilon = 0$.

Note that $\partial\pi^{f*}(\theta^* + \varepsilon)/\partial\varepsilon$ (at $\varepsilon = 0$) $= (1/8)(3\theta^l - \theta^u)(3\theta^u - 5\theta^l)$.

Therefore, if $(1/3)\theta^u < \theta^l < (3/5)\theta^u$, then it is positive. Thus, the feasible set of (F^l, P^l) so that the leader gets accommodating response can be given by

$\{(F^l, P^l) \mid (1/3)\theta^u < p^l < (3/5)\theta^u \text{ and } F^l = 0.5(\theta^l - P^l)^2\}$. ▲

Combining Propositions 4.1 and 4.3 , we get

Proposition 4.4

The price follower accommodates the price leader if and only if the latter serves the low demanders.

This result can be interpreted as a sort of price version of Salop and Gelman's (1983) "judo economics". In their paper, a price leader can manage an accommodating response by committing itself to a limited capacity. The leader's capacity limitation allows the follower to enjoy its monopoly power over the rest of the market. Thus, it might be profitable for the follower to accommodate the leader rather than going into a price war over the control of the whole market. In our model, the leader can get an accommodating response only by limiting itself to the low demanders. It does so by charging an unit price high enough to confine itself among the low demanders. Thus, the follower has the option of charging a high fixed fee by serving only the high demanders. Hence, it turn outs to be profitable for the follower to serve only the high demanders, rather than going into a price war over the control of the whole market.

Post entry price competition shows that under two-part pricing both the price leader and the price follower can coexist in equilibrium with positive profit. Thus, whether or not the entrant is a price leader or a follower, its post entry equilibrium profit is positive. Therefore, depending on the market parameters it is quite possible that the entrant's post entry profit is greater than the entry cost involved. In this case, the entrant's net payoff from entry is positive and entry occurs successfully. However, since the profit of the price follower is greater than that of the price leader, given the same level of entry cost, entry is more likely when the entrant is the price follower.

From Proposition 4.2 it follows that, if only the follower has the privilege of charging two-part price and the leader does not, still the latter can manage an accommodating response and earn a positive after entry profit. Thus, when the entrant is the price leader and the incumbent is the price follower, entry is facilitated even if only the incumbent has the option of charging a two-part price, whereas the entrant does not.

On one hand our results support the competitiveness of the market, but on the other hand they do not. In the traditional price competition with uniform pricing any positive amount of entry cost can create complete entry barriers. However, with two part pricing we have shown that this is not necessarily true. Here entry is blocked only if the entry cost is greater than the entrant's after entry equilibrium profit. Obviously, this depends on the amount of entry cost and also on the market parameters. On the other hand, we have shown that the firms can coexist in equilibrium with positive profits by serving different groups of consumers. This contrasts with the traditional price competition where two firms are sufficient to bring down profits to zero.

Section 4.5: Contestability Doctrine Under Two-part Pricing

In this section our objective is to examine a "perfectly contestable market" (Baumol, Panzar and Willig, 1982) under two part pricing. In a contestable market the incumbent is the price leader and the entrant is the price follower. It is also assumed that the incumbent is committed to a price schedule prior to the entry decision of the entrant. (This assumption about timing is different from that made in our model. In our model the choices of price schedules follow after the entry decision of the entrant.) Thus, if there is no entry cost involved, then under uniform pricing, contestability argues that only one potential entrant is enough to impose sufficient threat on the existing incumbent so that the latter will have to price at marginal cost and will earn zero profit. To make our model consistent with the contestability doctrine, we make two changes in assumptions. First, we omit the assumption of positive entry cost, so now entry is perfectly reversible with no cost involved. Secondly, now the incumbent is committed to a price schedule prior to the entry and subsequent choice of a price schedule by the entrant. It is shown that the contestability result does not hold under two part pricing. Referring back to Proposition 4.3, it follows that now the incumbent (price leader) can concentrate on the low demanders, so that if the entrant enters it will serve the high demanders, instead of undercutting it. In other words, now the incumbent can exist with positive profit by concentrating on the low demanders, because for the potential entrant it is more profitable to serve the high demanders than undercutting the incumbent. Thus, again the option of avoiding head to head price competition by serving different groups of consumers, makes the competition under two part pricing significantly different from that under uniform pricing. We close this section by summarizing the above discussion in the following proposition.

Proposition 4.5

> If the consumers differ in tastes, then under two part pricing, one potential entrant with zero entry cost is not sufficient to ensure zero profit for the incumbent.

Section 4.6: Competition with Simultaneous Price Setting

Throughout our analysis, we have assumed sequential price setting by the two firms. In this section we briefly discuss the consequence of a simultaneous price setting by the firms.

Simultaneous price setting with uniform prices is the standard Bertrand price competition, where the equilibrium price must equal to marginal cost. Thus, for the entrant, post entry equilibrium profit is zero and hence, entry never occurs. Therefore, with uniform pricing, the results under a simultaneous price setting vis-a-vis a sequential price setting remain unchanged. However, with two-part pricing, the outcomes under simultaneous price setting differ significantly from those under sequential price setting. Under simultaneous price setting both the firms should make equal profit in equilibrium. If not, then the firm which makes lower profit, can do better by duplicating the other's price schedule with an ε less in the fixed fee / unit price. This phenomenon leads us to Proposition 4.6 which shows that, under simultaneous price setting, the equilibrium fixed fee is zero and the equilibrium unit price is equal to the marginal cost.

Proposition 4.6

Under simultaneous price setting, price competition with two-part tariffs has a unique equilibrium as given by,

$\{(F^i = 0,\ p^i = 0),\ (F^j = 0,\ p^j = 0)\},\ i \neq j$

Proof: First note that $\{(F^i = 0,\ P^i = 0),\ (F^j = 0,\ P^j = 0)\}$ is an equilibrium. We will show that $\nexists$ any other equilibrium. Note that $F^i = F^j$ and $P^i \neq P^j$ can't be an equilibrium. If $P^i > P^j$, then firm i can do better by charging $P^i = P^j - \varepsilon$ and if $P^i < P^j$, then firm j can do better by charging $P^j = P^i - \varepsilon$. Similarly $F^i \neq F^j$ and $P^i = P^j$ can't be an equilibrium. Also note that $(F^i > F^j,\ P^i > P^j)$ and $(F^i < F^j,\ P^i < P^j)$ can't be equilibrium. Therefore, the remaining candidates for equilibrium are, $(F^i = F^j,\ P^i = P^j)$ with both F^i and P^i are not zero, $(F^i > F^j,\ P^i < P^j)$ and $(F^i < F^j,\ P^i > P^j)$. If $(F^i = F^j,\ P^i = P^j)$ with both F^i and P^i are not zero, then firm i can do better by lowering either F^i or P^i by ε, and thus, this can't be an equilibrium. Now consider $(F^i > F^j,\ P^i < P^j)$. If $\pi^i > \pi^j$, then firm j can do better by charging $F^j = F^i - \varepsilon$ and $P^j = P^i$. Similarly, if $\pi^i < \pi^j$, then firm i can do better. If $\pi^i = \pi^j$, then firm i can do better by charging $F^i = F^j - \varepsilon$ and $P^i = P^j$. By doing so, it gets firm j's whole market plus a portion of its original market. Similarly, $(F^i < F^j,\ P^i > P^j)$ can't be an equilibrium. This proves that $\{(F^i = 0,\ P^i = 0),\ (F^j = 0,\ P^j = 0)\}$ is the unique equilibrium. ▲

This proposition shows that with simultaneous price setting, the outcomes under uniform pricing vis-a-vis two-part tariffs become identical. Thus, under simultaneous price setting, the analysis of price competition with two-part tariffs does not add anything more to the standard Bertrand competition.

Section 4.7: Summary and Conclusions

This chapter has extended traditional price competition by incorporating two-part pricing as a strategic instrument. Given a positive entry cost entry is blocked under uniform price competition, since the after entry equilibrium profit comes down to marginal cost. However, entry is no longer blocked if we include two-part price as a strategic instrument. Given that consumers differ in tastes, two-part pricing allows the firms to avoid head to head competition by serving different segments of the market and to coexist with positive profit. Thus, it is possible that, after entry equilibrium profit is higher than the entry cost and entry occurs successfully.

In some sense, our model can also be interpreted in a locational framework. Suppose the firms are competing in uniform prices, however, there is another choice variable - location. The firms have the option to locate close to or away from the consumers' location. For example, one of the firms might be a factory outlet and the other is located in the shopping mall. Although the firms are competing in uniform prices, the consumer needs to pay a higher cost to go to the factory outlet than to the shopping mall. Then for the consumers, it is the same as choosing between two firms competing in two-part tariffs. Thus, it may be possible for the firms to charge prices greater than marginal costs and make positive profits in equilibrium.

Our analysis is based on some simplifying assumptions. We assumed linear demand (p = θ - q) for the consumers, zero cost of production and an uniform distribution of consumers' taste parameter θ. However, the idea of this paper goes through without these simplifying assumptions. The specific linear form of demand can be weakened by a non-crossing assumption. Allowing for cost functions with positive (constant) marginal cost of production does not change the results qualitatively. Given an uniform distribution of θ, the price leader serves the low demanders while the price follower serves the high demanders. However, this result might be changed with a positively skewed distribution of θ. One possible objection to our analysis might be that we have restricted both the price leader and the price follower to offer only one price schedule each. It can be argued that if the follower could offer two schedules, it would offer one for the low demanders (undercutting the leader) and one for the high demanders. This would enable it to control the whole market instead of sharing it with the price leader. However, in that case, the leader might as well come up with two schedules. Qualitatively, as long as both the firms are restricted to offer an equal number of price schedules. it is possible for the leader to manage an accommodating response. Moreover. if there is an increasing cost involved for offering an additional price schedule, it is possible to obtain a solution, where both the firms will offer only a limited and equal number of price schedules.

CHAPTER 5
REGULATING A MONOPOLIST WITH UNKNOWN DEMAND: AN ALTERNATIVE APPROACH

Section 5.1: Introduction

In regulated industries, the regulated firm often has better information about the demand for its product than does the regulator. In that case, a regulator faces two possible sources for information about demand: the regulated firm and the consumers in that industry. In the literature, Lewis and Sappington (1988) derive the optimal regulatory mechanism when the regulator depends exclusively on the firm for demand information. An alternative approach is taken in this chapter. Here the regulator solicits demand information only from the consumers. When the firm's marginal costs of production decline with output, significant gains are demonstrated.[1]

When the marginal costs of production decline with output, Lewis and Sappington's approach does not establish efficient prices. A single price is set regardless of demand. This procedure results in a price that exceeds the marginal cost of production for large demand and that falls short of marginal cost for small demand. The firm also commands rents under all but some intermediate levels of demand. It is shown here that the observed inefficiencies can be corrected if the regulator solicits demand information directly from the consumers. A regulatory mechanism is proposed which induces truthful revelation as a dominant strategy for every consumer. Furthermore, efficient prices are always implemented, the firm commands no rents from its superior information, and the regulator's budget is balanced in expectation.

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[1] For optimal regulatory policy without cost information, see Baron and Myerson (1982), Laffont and Tirole (1986), Sappington (1983), Sappington and Sibley (1988).

This chapter is organized as follows. Section 5.2 describes the model. Results are discussed in Section 5.3 and Section 5.4 concludes the chapter.

Section 5.2: Statement of The Model

The regulatory environment under consideration is the following. There are n consumers and a single producer. To focus on the case where inefficiencies arise in Lewis and Sappington's model, the firms marginal costs of production are assumed to decline with output i.e. $C'(q) > 0$ and $C''(q) < 0$.[2] It is common knowledge that demand for consumer i is given by $q^i = q(p, \theta_i)$, where p is the unit price and θ_i is consumer i's private information. θ_i's are drawn independently from a distribution with density function $f(\theta)$, defined over a nondegenerate interval $[\theta^l, \theta^u]$. It is assumed that $q_\theta(p, \theta) > 0$ and $q_p(p, \theta) < 0$. Also, the aggregate demand curve is assumed to be everywhere more steeply sloped than the firm's marginal cost curve for all realization of θ. Aggregate demand may or may not be known by the firm. Also, each individual's consumption is unobservable to the regulator.

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[2] Lewis and Sappington show that with upward sloping marginal costs of production the firm reveals its private information truthfully and first-best is always ensured by soliciting information from the firm.

The regulator is endowed with the power to establish a unit price p and collect a transfer payment T^i from consumer i. Prices and the transfer payments can be based on the consumers' reports of their private information.

Timing Of The Game

1. Consumer i learns θ_i, i = 1,...,n.
2. The regulator announces a pricing scheme $\{p(\theta_1',\ldots,\theta_n'), T^i(\theta_1',\ldots,\theta_n')\}$ i = 1,..,n.
3. All consumers simultaneously make reports of their private information about θ. Denote the report of consumer i (i = 1,...,n) by θ_i'.
4. Consumer i (i = 1,...,n) makes transfer payment T^i to the regulator.
5. The regulator announces p to the firm and gives a subsidy to the firm. The subsidy just covers the firm's total costs of production if it in fact produces and sells the quantity to satisfy reported aggregate demand at price p.
6. Production occurs. The firm is required to serve all realized demand at the regulated price.

Definition Of First-Best

As a benchmark, we record the first-best policy, which is the one the regulator would implement if she could observe the realizations of $(\theta_1,\ldots,\theta_n)$. The first-best maximizes expected consumers' surplus while ensuring the firm nonnegative profit.

The first-best policy consists of prices $p^*(\theta_1,\ldots,\theta_n)$, a total transfer payment $T^*(\theta_1,\ldots,\theta_n)$ from the regulator to the firm and a transfer payment $T^i(\theta_1,\ldots,\theta_n)$ from consumer i to the regulator (i = 1,...,n) which satisfy the following properties $\forall\ (\theta_1,\ldots,\theta_n)$:

$$(5.1)\quad p^*(\theta_1,\ldots,\theta_n) = C'(\sum_1^n q(p^*(\theta_1,\ldots,\theta_n),\theta_i));$$

$$(5.2)\quad T^*(\theta_1,\ldots,\theta_n) = C(\sum_1^n q(p^*(\theta_1,\ldots,\theta_n),\theta_i)) - p^*(\theta_1,\ldots,\theta_n)(\sum_1^n q(p^*(\theta_1,\ldots,\theta_n),\theta_i));$$

$$(5.3)\quad \sum_1^n T^i(\theta_1,\ldots,\theta_n) = T^*(\theta_1,\ldots,\theta_n); \quad \text{and}$$

$$(5.4)\quad \int_{p^*(\theta_1,\ldots,\theta_n)}^{\infty} q(\xi,\ \theta_i)d\xi \ \geq T^i(\theta_1,\ldots,\theta_n), \quad i = 1,\ldots,n.$$

(5.1) implements marginal cost prices, (5.2) holds the firm to zero profit and (5.4) ensures that T^i's are sufficiently small that all consumers enjoy a nonnegative consumer surplus. A balanced budget constraint is explicitly introduced through (5.3). Since social costs may arise if the regulator's budget is not balanced, (5.3) is considered as a part of first-best.

Definitions

Define constraints (M), (B) and (T) as follows.

$$(M)\quad p(\theta_1',\ldots,\theta_n') = C'(\sum_1^n q(p(\theta_1',\ldots,\theta_n'),\theta_i'))$$

$$(B)\quad \sum_1^n T^i(\theta_1',\ldots,\theta_n') = C(\sum_1^n q(p(\theta_1',\ldots,\theta_n'),\theta_i')) - p(\theta_1',\ldots,\theta_n')(\sum_1^n q(p(\theta_1',\ldots,\theta_n'),\theta_i'))$$

$$(T)\quad S(\theta_i|(\theta_1',..,\theta_i,..,\theta_n')) \geq S(\theta_i'|(\theta_1',..,\theta_i,..,\theta_n'))$$

$$\forall\ \theta_i,\ \theta_i' \text{ and } \theta_j'\ \ i \neq j,\ \ i,\ j = 1,\ldots,n$$

where $S(\theta_i' | (\theta_1', .., \theta_i, ..., \theta_n')) = \int_{p(\theta_1', .., \theta_i', ..., \theta_n')}^{\infty} q(\xi, \theta_i) d\xi - T^i(\theta_1', .., \theta_i' ..., \theta_n')$.

In words, $S(\theta_i' | (\theta_1', .., \theta_i, .., \theta_n'))$ is the net surplus of individual i if he is of type θ_i and reports θ_i', while other consumers report $(\theta_1', .., \theta_{i-1}', \theta_{i+1}', ..., \theta_n')$. The first constraint (M) is marginal cost pricing. Constraint (B) is a balanced budget constraint. Constraint (T) ensures that truth telling is a dominant strategy for every consumer. It is obvious from (5.1) - (5.4) that if a scheme satisfies (M), (B) and (T), then it sustains the first best policy as a unique Nash equilibrium.

Section 5.3: Results and Discussion

The next two propositions illustrate special cases where the first-best policy can always be ensured.

Proposition 5.1

If there is only one consumer in the market (n = 1), then the scheme

$p(\theta') = C'(q(p(\theta'), \theta'))$ and

$T(\theta') = C(q(p(\theta'), \theta') - p(\theta')q(p(\theta'), \theta')$

satisfies (M), (B) and (T), and hence, achieves the first-best.

Proof: The consumer chooses θ' to maximize $L = \int_{p(\theta')}^{\infty} q(\xi, \theta) d\xi - T(\theta')$

where $T(\theta') = \int_{p(\theta')}^{\infty} C'(\xi) d\xi - p(\theta')q(p(\theta'), \theta')$ and $C'(q(p(\theta'), \theta')) = p(\theta')$

Now $(\partial L/\partial\theta') = - q(p(\theta'), \theta)(\partial p(\theta')/\partial\theta') - (\partial T(\theta')/\partial\theta')$

However, $(\partial T(\theta')/\partial\theta')$

$= \{C'(q(p(\theta'), \theta'))[(\partial q(p(\theta'), \theta')/\partial\theta') + (\partial q(p(\theta'), \theta')/\partial p)((\partial p(\theta')/\partial\theta')]$

$- p(\theta')[(\partial q(p(\theta'), \theta')/\partial\theta') + (\partial q(p(\theta'), \theta')/\partial p)((\partial p(\theta')/\partial\theta')]$

$- q(p(\theta'), \theta')(\partial p(\theta')/\partial\theta')\}$

$= [C'(q(p(\theta'), \theta')) - p(\theta')][(\partial q(p(\theta'), \theta')/\partial\theta')$

$+ (\partial q(p(\theta'), \theta')/\partial p)((\partial p(\theta')/\partial\theta')] - q(p(\theta'), \theta')(\partial p(\theta')/\partial\theta')$

$= - q(p(\theta'), \theta')(\partial p(\theta')/\partial\theta')$ [since $C'(q(p(\theta'), \theta')) = p(\theta')$]

$\Rightarrow (\partial L/\partial\theta') = [q(p(\theta'), \theta') - q(p(\theta'), \theta)](\partial p(\theta')/\partial\theta')$

$\Rightarrow (\partial L/\partial\theta')$ (at $\theta' = \theta$) $= 0$

This implies $\theta' = \theta$ satisfies the first order condition for maximization. It is straight forward to check that the second order condition is also satisfied. ▲

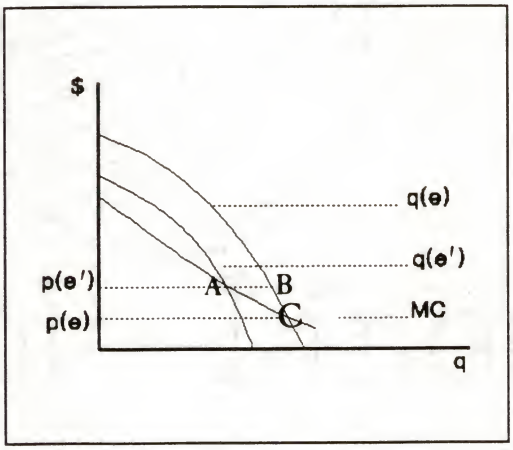

Figure 5.1

The intuition behind Proposition 5.1 is as follows. Given the reported demand, the scheme establishes marginal cost pricing and requires the consumer to make a transfer payment equal to the total cost less the revenue. Now, if the consumer understates his demand, then because of the downward sloping marginal cost curve, he is required to pay a higher price but is charged a lower transfer payment. In Figure 5.1, price $p(\theta')$ is established if he reports $\theta' < \theta$ and $p(\theta)$ is established if he reports truthfully. However, starting from $p(\theta)$, an increase in price lowers gross consumer surplus (consumer surplus without considering the transfer payment) which is measured along the true demand curve, whereas the decrease in transfer payment is measured along the marginal cost curve. In Figure 5.1, area $[p(\theta')BCp(\theta)]$ represents the decrease in gross consumer surplus and area $[p(\theta')ACp(\theta)]$ represents the decrease in transfer payment. Since for $p \geq p(\theta)$ the true demand curve lies everywhere above the marginal cost curve, the decrease in gross consumer surplus is more than the decrease in transfer payment. In Figure 5.1, he loses area [ABC] by reporting $\theta' > \theta$. On the other hand, if the consumer overstates his demand, then he pays a lower price but is required to make a higher transfer payment. Since for $p \leq p(\theta)$ the true demand curve lies everywhere below the marginal cost curve, the increase in gross consumer surplus due to a lower price is less than the increase in transfer payment. As a result, the consumer loses by misreporting his demand.[3]

Proposition 5.2

Given there is more than one consumer, for any binary distribution of θ, the scheme

$$p(\theta_1',\ldots,\theta_n') = C'\Big(\sum_1^n q(p(\theta_1',\ldots,\theta_n'),\theta_i')\Big) \text{ and } T^i(\theta_1',\ldots,\theta_n') =$$

$$(1/n)\Big\{C\Big(\sum_1^n q(p(\theta_1',\ldots,\theta_n'),\theta_i')\Big)$$

$$- \; p(\theta_1',\ldots,\theta_n')\Big(\sum_1^n q(p(\theta_1',\ldots,\theta_n'),\theta_i')\Big)\Big\}$$

---

[3] Proposition 5.1 depends on the assumption that marginal costs of production decline with output. With upward sloping marginal costs of production the consumer gains by misreporting his demand.

satisfies (M), (B) and (T), and hence sustains the first-best as a unique Nash equilibrium.

Proof: Note that the scheme satisfies (M) and (B) by construction. Thus it is enough to show that the scheme also satisfies (T). Let θ take two possible values θ^l and θ^h with some probabilities. Suppose n^l number of people report θ^l and n^h number of people report θ^h ($n^h+n^l = n-1$). Now consider any one of the remaining individuals. He can either be of type θ^l or θ^h.

Case 1: The individual is of type θ^l.

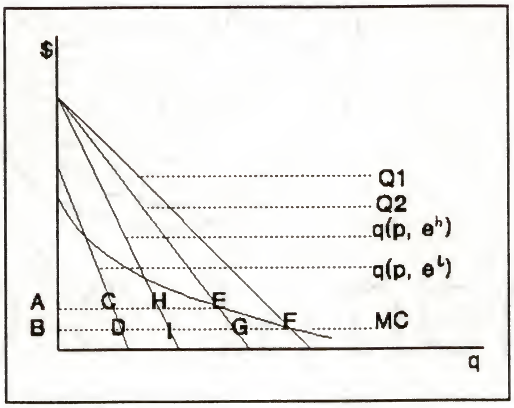

Figure 5.2

In Figure 5.2, Q1 denotes the reported aggregate market demand if he reports θ^h and Q2 denotes reported aggregate market demand if he reports truthfully. If he reports he is of θ^h, then he gets a lower price and his gain in gross surplus = [ACDB]. However, he needs to pay a higher transfer payment and his loss in transfer payment = (1/n) [AEFB].

Now, $(1/n)[AEFB] > (1/n)[AEGB] = (1/n)[(n^l+1)[ACDB] + n^h[AHIB]]$

$> (1/n)[(n^l+1)[ACDB] + n^h[ACDB]] = [ACDB]$

Thus, he is worse off by not telling the truth.

Case 2: The individual is of type θ^h.

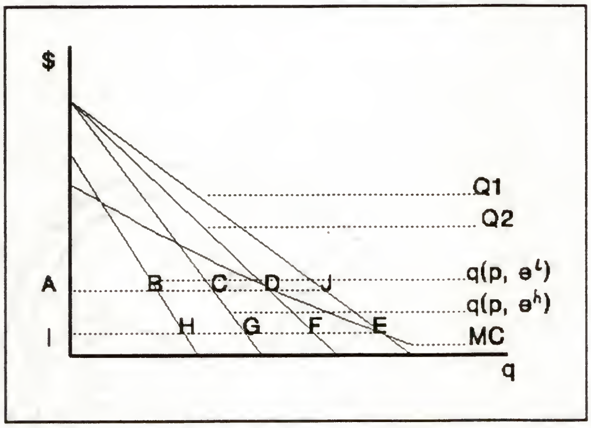

Figure 5.3

In Figure 5.3, Q1 denotes reported aggregate market demand if he reports truthfully and Q2 denotes reported aggregate market demand if he reports θ^l. If he reports that he is of type θ^l then he gains in transfer payment, but he needs to pay a higher price. His loss in gross surplus = [ACGI]. Gain in transfer payment = (1/n)[ADEI].

Now, $(1/n)[ADEI] < (1/n)[AJEI] = (1/n)\{n^l[ABHI] + (n^h+1)[ACGI]\}$

$< (1/n)\{n^l[ACGI] + (n^h+1)[ACGI]\} = [ACGI]$.

Thus, he is worse off by not telling the truth.

=> The scheme satisfies (T). ▲

In the scheme described in Proposition 5.2, $T^i = T^j$, i, j = 1...,n. That is, each consumer pays the same transfer payment. Thus, first-best is achieved through a non-discriminatory pricing policy. This is an advantage of the scheme if legal restrictions prohibit discriminatory pricing.

The intuition behind Proposition 5.2 is as follows. Let $\theta \in \{\theta^l, \theta^h\}$, $\theta^l < \theta^h$. If a consumer of type θ^l overstates his demand and claims to be of θ^h, then because the marginal cost is downward sloping, a lower price is established. Accompanying the lower price is a higher total transfer payment by consumers. Following the intuition that underlies Proposition 5.1, the total increase in transfer payment outweighs the total increase in gross consumer surplus. Thus, the average increase in transfer payment is greater than the average increase in gross consumer surplus. Since the total transfer payment is distributed equally among the consumers, the transfer payment by each individual is increased exactly the average increase. However, since $\theta^l < \theta^h$, the increase in consumer surplus for an individual of type θ^l is less than the average increase; therefore the increase in his surplus is less than the increase in his transfer payment. On the other hand, if a consumer of type θ^h understates his demand, then the average decrease in gross consumer surplus outweighs the average decrease in transfer payment. With $\theta^h > \theta^l$, the decrease in the consumer surplus of an individual of type θ^h exceeds the average decrease: therefore the decrease in gross consumer surplus for the individual outweighs the decrease in the transfer payment he is required to make.

It remains to determine whether the first-best policy can be ensured for more general distributions of θ. For example, consider a discrete distribution of θ where $\theta \in \{\theta^1, \ldots, \theta^m\}$, $m > 2$, and $\theta^i > \theta^j$, $\forall\ i > j$. Truth telling remains a dominant strategy for the extreme (θ^1 or θ^m) consumers under a non-discriminatory policy. If a consumer overstates his demand, then the average increase in transfer payment is more than the average increase in gross consumer surplus. Since $\theta^1 < \theta^i$, $\forall$ i = 2,...,m, for a consumer of type θ^1, his individual increase in consumer surplus is less than the average increase, and thus, he loses by overstating demand. On the other hand, if a consumer understates his demand, then the average decrease in gross consumer surplus is more than the average decrease in transfer payment. Since $\theta^m > \theta^i$, $\forall$ i = 2,...,m-1, his individual decrease in gross consumer surplus is more than the average decrease and thus, he loses by understating demand. However, for consumers with intermediate θ, the individual change in gross consumer surplus cannot be compared to the average change without knowing the reports of the others. Since a dominant strategy requires truth telling to be optimal irrespective of the reports of the others, for consumers with intermediate θ truth telling may no longer a dominant strategy under a non-discriminatory pricing policy.

Since the first-best policy cannot be ensured through a non-discriminatory pricing policy, we look for a discriminatory pricing policy (allowing $T^i \neq T^j$) which can ensure first-best policy for more general distributions of θ. For any continuous distribution of θ, the following scheme "S" induces truthful revelation as a dominant strategy for every consumer and first-best prices are always achieved.

Definition Of Scheme "S"

Define a scheme "S" as,

$$p(\theta_1', \ldots, \theta_n') = C'(\sum_1^n q(p(\theta_1', \ldots, \theta_n'), \theta_i'))$$

$T^i(\theta_1', \ldots, \theta_n') = A_i$ + a constant K (independent of $(\theta_1', \ldots, \theta_n')$).

K is sufficiently small that all consumers receive nonnegative surplus in equilibrium and

$$A_i = \int_0^{\{\sum_1^n q(p(\theta_1', \ldots, \theta_n'),\ \theta_i')\}} C'(\xi)d\xi - \int_{p(\theta_1', \ldots, \theta_n')}^{\infty} \sum_{j \neq i} q(\xi,\ \theta_j')d\xi$$

In words, A_i is the total variable costs required to satisfy the aggregate reported demand (including consumer i) at price $p(\theta_1', \ldots, \theta_n')$, less the aggregate gross consumer surplus without consumer i.

Lemma 5.1

For any continuous distribution of θ, the scheme S satisfies (M) and (T).

Proof: The scheme S satisfies (M) by construction. Thus, it is enough to show S also satisfies (T). Consider individual i. He can misreport θ_i by either reporting $\theta_i > \theta_i$, or $\theta_i < \theta_i$.

Case 1 $\theta_i' > \theta_i$

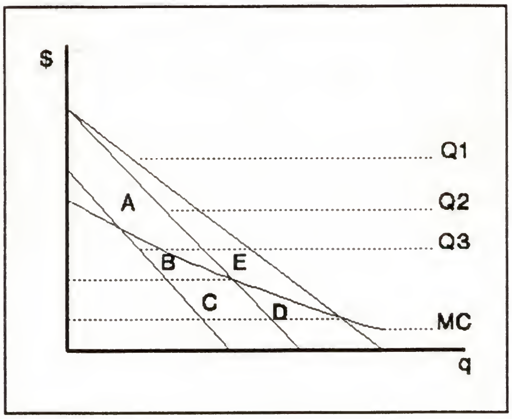

Figure 5.4

In Figure 5.4, Q1 denotes aggregate reported market demand including i, if he reports $\theta_i' > \theta_i$, Q2 denotes aggregate reported market demand including i, if he reports $\theta_i' = \theta_i$ and Q3 denotes aggregate reported market demand without i.

His net surplus if he tells the truth = [A+B] - [B] - K = [A] - K. If he reports $\theta_i > \theta_i$, his net surplus = [A+B+C] - [B+C+D] - K = [A-D] - K. Therefore he is worse off by reporting $\theta_i > \theta_i$.

Case 2 $\theta_i' < \theta_i$

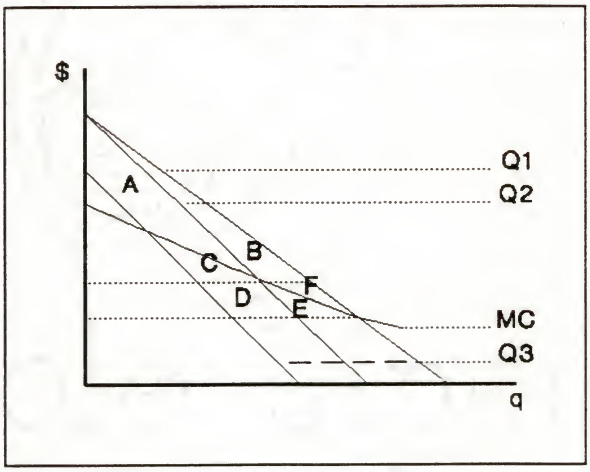

Figure 5.5

In Figure 5.5, Q1 denotes aggregate reported market demand including i, if he reports $\theta_i' = \theta_i$, Q2 denotes aggregate reported market demand including i, if he reports $\theta_i' < \theta_i$ and Q3 denotes aggregate reported market demand without i.

His net surplus if he tells the truth

= [A+B+C+D+E+F] - [C+D+E] - K = [A+B+F] -K.

His net surplus if he reports $\theta_i < \theta_i$ = [A+B+C] - [C] - K = [A+B] - K.

Therefore, he is worse-off by reporting $\theta_i < \theta_i$.

=> The scheme satisfies (T). ▲

The scheme "S" however, does not satisfy (B), that is, the regulator's budget may not be balanced. This raises the question, can a different scheme be found that balances the regulator's budget ? Balanced budget requires $\Sigma T^i = T$. Thus, given $(T^1, \ldots, T^{n-1})$ it automatically defines T^n. This reduces a degree of freedom for the regulator so that she can now choose only (n - 1) of the T^i independently. For a continuous distribution of θ, its turns out that under some assumptions on the transfer payments, choosing (n - 1) of the T^i 's is not sufficient to ensure truth telling for n consumers.

Proposition 5.3

For a continuous distribution of θ, if the individual transfer payment functions are twice continuously differentiable, then there is no scheme which satisfies (M), (B) and (T) simultaneously.

Proof: We present this proof for n = 2. It is also assumed that . The proof is by contradiction. Suppose $\exists$ a scheme S which satisfies (M), (B) and (T) simultaneously. Then from (T)

$$\theta_i = \arg\max \{\int_{p(\theta_i', \theta_j)}^{\infty} q(\xi, \theta_i)d\xi - T^i(\theta_i', \theta_j)\} \ \forall \ \theta_i, \theta_j \in [\theta_l, \theta_u], \ i \neq j = 1.\ 2$$

where, $p(\theta_i', \theta_j) = C'(q(p(\theta_i', \theta_j), \theta_i') + q(p(\theta_i', \theta_j), \theta_j))$ by [(M)]

and $T^i(\theta_i', \theta_j) = T(\theta_i', \theta_j) - T^j(\theta_i', \theta_j)$ by [(B)]

$$\text{Let } L = \int_{p(\theta_i', \theta_j)}^{\infty} q(\xi, \theta_i)d\xi - T^i(\theta_i', \theta_j)$$

Then $(\partial L/\partial \theta_i')$ (at $\theta_i' = \theta_i$) = 0 [from first order condition]

Now, $(\partial L/\partial\theta_i') = -\, q(p(\theta_i', \theta_j), \theta_i')(\partial p(\theta_i', \theta_j)/\partial\theta_i') - (\partial T^i(\theta_i', \theta_j)/\partial\theta_i')$

$= -\, q(p(\theta_i',\theta_j), \theta_i')(\partial p(\theta_i', \theta_j)/\partial\theta_i') + (\partial T^j(\theta_i', \theta_j)/\partial\theta_i') - (\partial T(\theta_i', \theta_j)/\partial\theta_i')$

However, $(\partial T(\theta_i', \theta_j)/\partial\theta_i') = (\partial\{\int_0^{q^*} C'(\xi)d\xi - p(\theta_i', \theta_j)q^*\}/\partial\theta_i')$

where $q^* = q(p(\theta_i', \theta_j), \theta_i') + q(p(\theta_i', \theta_j), \theta_j)$

$\Rightarrow (\partial T(\theta_i', \theta_j)/\partial\theta_i')$

$= C'(q^*)(\partial q^*/\partial\theta_i') - p(\theta_i', \theta_j)(\partial q^*/\partial\theta_i') - q^*(\partial p(\theta_i', \theta_j)/\partial\theta_i')$

$= [C'(q^*) - p(\theta_i', \theta_j)](\partial q^*/\partial\theta_i') - q^*(\partial p(\theta_i', \theta_j)/\partial\theta_i')$

$= -\, q^*(\partial p(\theta_i', \theta_j)/\partial\theta_i')$

$\Rightarrow (\partial L/\partial\theta_i') = q(p(\theta_i', \theta_j), \theta_i')(\partial p(\theta_i',\theta_j)/\partial\theta_i') + (\partial T^j(\theta_i', \theta_j)/\partial\theta_i')$

Since $(\partial L/\partial\theta_i')$ (at $\theta_i' = \theta_i$) $= 0$, we have

$$q(p(\theta_i, \theta_j), \theta_j)(\partial p(\theta_i, \theta_j)/\partial\theta_i) + (\partial T^j(\theta_i, \theta_j)/\partial\theta_i) = 0 \ \forall\ \theta_i, \theta_j \in [\theta_l, \theta_u] \qquad (5.5)$$

Similarly considering individual j, we get

$$q(p(\theta_i,\theta_j), \theta_i)(\partial p(\theta_i,\theta_j)/\partial\theta_j) + (\partial T^i(\theta_i, \theta_j)/\partial\theta_j) = 0 \ \forall\ \theta_i, \theta_j \in [\theta_l, \theta_u] \qquad (5.6)$$

Also we have $(\partial T^i/\partial\theta_i) + (\partial T^j/\partial\theta_i) = (\partial T/\partial\theta_i)$

$$= -(q(p(\theta_i, \theta_j), \theta_i) + q(p(\theta_i, \theta_j), \theta_j)) \qquad (5.7)$$

From (5.5) and (5.7), we get

$(\partial T^i/\partial\theta_i) = -\, q(p(\theta_i, \theta_j), \theta_i)(\partial p(\theta_i, \theta_j)/\partial\theta_i)$

$$\Rightarrow (\partial^2 T^i/\partial\theta_i\partial\theta_j) = -\, [q(p(\theta_i, \theta_j), \theta_i)(\partial^2 p(\theta_i, \theta_j)/\partial\theta_i\partial\theta_j)]$$

$$-\, [(\partial p(\theta_i, \theta_j)/\partial\theta_i)(\partial q(p(\theta_i, \theta_j), \theta_i)/\partial p)(\partial p(\theta_i, \theta_j)/\partial\theta_j)] \qquad (5.8)$$

Also from (5.6), we get

$(\partial^2 T^i/\partial\theta_i\partial\theta_j)$

$= -\, [q(p(\theta_i, \theta_j), \theta_i)(\partial^2 p(\theta_i, \theta_j)/\partial\theta_i\partial\theta_j)]$

$-\, [(\partial p(\theta_i, \theta_j)/\partial\theta_j)\{(\partial p(\theta_i, \theta_j)/\partial\theta_i)(\partial q(p(\theta_i, \theta_j), \theta_i)/\partial p)$

$$+\, (\partial q(p(\theta_i, \theta_j), \theta_i)/\partial\theta_i)\}] \qquad (5.9)$$

Combining (5.8) and (5.9), we get $(\partial p(\theta_i, \theta_j)/\partial \theta_j)(\partial q(p(\theta_i, \theta_j), \theta_i)/\partial \theta_i) = 0$ This is a contradiction, since $\forall\ \theta_i, \theta_j \in [\theta_l, \theta_u]$, both terms are nonzero, and hence, their product cannot be zero. ▲

The intuition behind this proposition is as follows. Consider two consumers i and j with private information θ_i and θ_j respectively. Suppose j reports $\theta_j' > \theta_j$. Then i may have an incentive to report $\theta_i' < \theta_i$. Such a report brings the reported aggregate demand closer to true aggregate demand, thereby increasing total surplus. It also increases j's share of the total transfer payment, since i's share of this payment is the first best level less j's share. Therefore, i might gain by under reporting demand; so truth telling is no longer a dominant strategy. Intuitively, the imposition of a balanced budget constraint provides room for "free riding." Since an individual's transfer payment is the first best level of transfer payment less the transfer payments of the others, he can decrease his share of payment and increase the share of others by misreporting his demand. In scheme "S", the free rider's problem could be avoided because balanced budget constraint was not imposed.

Proposition 5.3 is similar to Hurwicz's (1975) result in the context of public goods. He showed it is not possible to have any mechanism which ensures efficient output of the public good, makes truth telling a dominant strategy and balances budget simultaneously. The scheme "S" is similar to the demand revealing tax scheme developed by Clarke (1971) in the context of public goods. A demand revealing tax scheme ensures efficient output of the public good and makes truth telling a dominant strategy. However, as in "S", the budget is not necessarily balanced under the demand revealing scheme.[4]

Although the regulator's budget is not necessarily balanced for all realizations of $(\theta_1, \ldots, \theta_n)$, it is possible to balance the budget in expectation (that is, $E_\theta(T^*) = E_\theta(\Sigma\, T^i)$). Lemma 5.3 defines "K" so that the budget is balanced in expectation. Proposition 5.4 follows immediately from Lemma 5.2 and Lemma 5.3.

Lemma 5.3

If $K = (1/n)\{\int \ldots \int [T^*(\theta_1, \ldots, \theta_n) - \sum_1^n A^i(\theta_1, \ldots, \theta_n)]\, f(\theta_1) \ldots f(\theta_n) d\theta_1 \ldots d\theta_n\}$,

then the scheme S sustains the first-best level of transfer payment $T^*(\theta_1, \ldots, \theta_n)$ in expectation, that is, $E_\theta(T^*) = E_\theta(\sum_1^n T^i)$.

Proof : Follows directly by computing expectations.

Proposition 5.4

Under the scheme S, ∃ a unique Nash equilibrium where the first-best price level $p^*(\theta_1, \ldots, \theta_n)$ is achieved for every realization of $(\theta_1, \ldots, \theta_n)$ and the first-best level of transfer payment $T^*(\theta_1, \ldots, \theta_n)$ is collected from consumers in expectation ($E_\theta(T^*) = E_\theta(\sum_1^n T^i)$).

Section 5.4: Summary and Conclusions

Optimal regulatory policies have been designed in a setting where the firm has superior information about consumer demand. In the literature, Lewis and Sappington derive the optimal regulatory mechanism when the regulator depends exclusively on the firm for demand information. We have allowed the regulator to collect information directly from the consumers and have demonstrated significant gains. When marginal costs of production decline with output, Lewis and Sappington's approach does not establish efficient prices, whereas the present approach does so for every realization of demand. Also, in contrast to Lewis and Sappington's approach, the firm does not command any rent from its superior demand information under the regulatory mechanism proposed here. Furthermore, the regulator's budget is balanced in expectation under our plan.

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[4] In the context of the public goods, d'Aspremont and Gerard-Varet (1979) propose a mechanism which balance the budget for all realizations of $(\theta_1, \ldots, \theta_n)$. However, under their mechanism truthful revelation may not be a dominant strategy.

Combining information from both the firm and the consumers is an interesting possibility to achieve the first-best. For example, if both the firm and the consumers are asked to report their private information, and production occurs only if they match, then first-best can be achieved as a Nash equilibrium. However under this scheme, there are other equilibria where first-best is not achieved. Therefore, attention should be focused on schemes which induce the outcome as a unique Nash equilibrium.

CHAPTER 6
CONCLUSIONS

This dissertation has examined strategic interactions among agents in an industry when each has limited market power. Four different scenarios have been considered each each essay of this dissertation has discussed one such scenario. In the first three essays, each firm has limited market power because of small number of firms in the industry. Moreover, in the first and second essay, sequential choices of inputs and outputs have provided additional room for strategic behavior. In the fourth essay, the firm has limited market power through its superior information about demand. It has turned out that the firms command rents from their market power in the first three scenarios, whereas in the fourth essay, the firm commands no rents from its superior information.

Each scenario discussed in this dissertation has scope for further research. I conclude this chapter by briefly summarizing these essays and outlining the scope for future research.

The first essay has analyzed the scope for strategic behavior with sequential input choice. When the inputs are perfect substitutes, it is shown that a "first mover advantage" may make commitment to a relatively expensive input in the first-stage profitable for one firm. The second firm follows by producing in the second-stage with relatively inexpensive input.

In the case of perfect substitutes the "first mover advantage" is the basis for strategic behavior. However, this "first mover advantage" remains valid for any additively separable production function, or more generally, whenever the isoquants touch the axes. Therefore, it would be interesting to analyze a general production function, where the isoquants are convex but touch the axes, so that "cost complementarity" and "first mover advantage" jointly provide the basis for strategic behavior. Since, "first mover advantage" is associated with asymmetric "leader-follower" equilibria, while a symmetric pure strategy equilibrium is likely to exist in the case of "cost complementarity," it is possible that, for a general neo-classical technology, the two different basis for strategic behavior might interact in such a way that a symmetric solution does not exist even in a symmetric model.

The second essay endogenizes the order of moves in duopoly games. For $c_1 < c_2$ and $c_1 > c_1^* > c_2$, the model generates the Cournot-Nash equilibria as the unique SPNE of the game. However, in the former case the firms produce their entire output simultaneously in the first period, whereas in the latter case they produce only in the second period. Stackelberg outcomes are generated endogenously for $c_2 < c_1 < c_1^*$. In this case, one of the firms behaves as a leader and produces its entire output in the first period. The other behaves as a follower and produces its best response in the second period. However, for $c_1 = c_2$ and $c_1 = c_1^*$, we encounter multiple SPNE, where both Cournot-Nash and Stackelberg outcomes can result endogenously from the model.

In this essay I have considered a symmetric model. It would be interesting to analyze an asymmetric model, since in the case of Stackelberg solution, an asymmetricity in the production functions may dictate who will be Stackelberg leader and who will be the follower.

The third essay considers competition in two-part tariffs. It is shown that under two-part tariffs with sequential price setting, the firms coexist in equilibrium with positive profit by serving different segments of the market. It is also shown that entry is facilitated under two-part tariffs.

This analysis is based on some simplifying assumptions. I have assumed linear demands ($p = \theta - q$) for the consumers, zero cost of production and a uniform distribution of consumers' taste parameter θ. However, the idea of this paper goes through without these simplifying assumptions. The specific linear form of demand can be weakened by a non-crossing assumption. Allowing for cost functions with positive (constant) marginal cost of production does not change the results qualitatively. Given a uniform distribution of θ, the price leader serves the low demanders while the price follower serves the high demanders. However, this result might be changed with a positively skewed distribution of θ.

The fourth essay has designed optimal regulatory policies in a setting where the firm has superior information about consumer demand. In the literature, Lewis and Sappington derive the optimal regulatory mechanism when the regulator depends exclusively on the firm for demand information. I have allowed the regulator to collect information directly from the consumers. When marginal costs of production decline with output, Lewis and Sappington's approach does not establish efficient prices, whereas the present approach does so for every realization of demand. Also, in contrast to Lewis and Sappington's approach, the firm does not command any rent from its superior demand information under the regulatory mechanism proposed here.

It has been assumed that θ's are drawn independently. It is interesting to explore the outcomes if this assumption is relaxed. This would make the model more realistic. However, that would be at the cost of more analytic complications.

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BIOGRAPHICAL SKETCH

Debashis Pal was born on January 9, 1964 in Calcutta, India. After graduating from Rama Krishna Mission Vidyalaya, Calcutta, in 1981, he earned his Bachelor of Statistics and Master of Statistics from Indian Statistical Institute, Calcutta, in 1984 and 1986 respectively. Thereafter he joined the graduate program of the Department of Economics, University of Florida, in Fall 1986. He earned his Master of Arts in economics in 1989. He graduated with his Ph.D. from the University of Florida in Summer 1990.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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